REALE ACCADEMIA DELLE SCIENZE DI TORINO

Plana II-22

ACCADEMIA
R DELLE SCIENZE
DI TORINO

PROPRIETATES

ALGEBRAICARUM CURVARUM.

primum edita Anno 1762–

AB
EDVARDO WARING, M.D.
MATHESEOS PROFESSORE LUCASIANO, CANTAB.
REGIÆ SOCIETATIS,
ET
BONONIENSIS SCIENTIARUM ACADEMIÆ SOCIO.

CANTABRIGIÆ,
TYPIS ACADEMICIS EXCUDEBAT J. ARCHDEACON.
Veneunt apud J. Woodyer, Cantabrigiæ; J. Beecroft, T. Payne, et T. Cadell, Londini; et D. Prince, Oxoniæ.
M. DCC. LXXII.

REVERENDO ADMODUM IN CHRISTO PATRI

EDMUNDO

EPISCOPO CARLIOLENSI,

NECNON

COLLEGII SANCTI *PETRI*

IN

ACADEMIA *CANTABRIGIENSI*

PRÆFECTO;

PRO

SUMMO SUO

IN CONFERENDO PROFESSORIO MUNERE

SUFFRAGII PATROCINIIQUE

BENEFICIO;

PROQUE ILLA, QUA DIU USUS EST,

OPTIMI FILII OPE ET AMICITIA;

HAS IN RE MATHEMATICA LUCUBRATIONES

GRATITUDINIS ERGO

D.D.D.

EDVARDUS WARING.

P R Æ F A T I O.

EUCLIDIS, Archimedis, Apollonii, Theodoſii, Pappii, *&c. Inventa notiora ſunt, quam ut quâquam egeant recenſione; & recentiorum innumeros labores hic ſcribendi modus non recipit: Haud tamen ab re erit, pauca, quæ ad hoc opus maxime pertinent, enarrare.* Adrianus Romanus *propoſuit æquationem multarum dimenſionum* Vietæ *reſolvendam, qui ejus reſolutionem deduxit ex eo quod ita conſtructa fuit hæc æquatio ab* Adriano, *ut ejus radix ſit chorda arcûs circuli, qui ſit ſubmultiplex arcûs, cujus datur chorda.* D'Moivre, *ex iiſdem principiis, reſolutionem æquationis* $ny + n.\frac{1-n^2}{2\cdot 3}y^3 + n.\frac{(1-n^2)(9-n^2)}{2\cdot 3\cdot 4\cdot 5}y^5 + \&c. = a$ *conſecutus eſt: deduxit etiam duas æquationes* $l = \frac{1+z^{2n}}{2z^n}$ & $x = \frac{1+z^2}{2z}$; *ubi* l & x *reſpective denotant coſinus arcuum, qui habent inter ſe rationem* n : 1; *exinde* $z^2 - 2xz + 1$ *erit diviſor quantitatis* $z^{2n} - 2lz^n + 1$; *conſequenter ex datis conſinibus* l & x *obtineri poſſunt radices æquationis* $z^{2n} - 2lz^n + 1 = 0$. Coteſius *diviſit circulum in* 2 λ *æquales partes; &, ſi modo ab omnibus diviſionibus ad punctum quodvis* P *ſitum in* OA *radio (ſi opus ſit producto) ducantur rectæ* AP, BP, CP, DP, EP, *&c. deinde ponantur* OA = a, & OP = x; *contentum ſub omnibus* AP, CP, EP, *&c. ſumptis a diviſionibus alternis per integrum circuitum adæquabit* $a^\lambda - x^\lambda$ *vel* $x^\lambda - a^\lambda$, *prout* P *fuerit intra vel extra circulum; & contentum ſub reliquis* BP, DP, *&c. in locis reliquis alternis erit* $x^\lambda + a^\lambda$: *Hæc proprietas aliquantulum generalior in ſequenti tractatu redditur; ſcilicet linea* OP *ne per punctum* A *at per punctum* H *tranſeat; ſit chorda arcûs* $(n \times AH - \overline{n-1} \times 180^\circ) = A$; *tum (ſi* λ *ſit impar numerus) erit contentum* $PA^2 \times PC^2 \times PE^2 \times \&c. = x^{2\lambda} + (A^2 - 2a^2)\,a^{\lambda-2}\,x^\lambda + a^{2\lambda}$; *&c.: huic adjiciuntur conſimilia; E.g. Sit æquatio* $x^2 - px + 1 = 0$, *cujus radices ſint* α & β, & *ſit* p *chorda arcûs* $180^\circ - \alpha$; *tum erit arcûs* $(180^\circ - na)$ *chorda* $= \alpha^n + \beta^n$.

a *Noſtras*

Noſtras Neale *primus invenit curvam, viz. ſemicubicalem parabolam, cujus peripheria æqualis eſt rectæ lineæ:* Torricelli *anno* 1644 *evulgavit cycloidem eſſe areæ circuli genitoris triplam:* Chriſtophorus Wren *eam rectificavit:* Huygenius *ejus involutam detexit:* Walliſius, *&c. plures ejus proprietates deduxere.* Roemer *invenit epicycloidem:* Caſwell, *e quibuſdam ejus proprietatibus a domino* D' la Hire *excogitatis, invenit quadraturam epicycloidis:* Neutonus *eam rectificavit, & ejus involutam detexit.*

Hic forſan haud abs re erit nominare conchoidem Nichomedis, *ciſſoidem* Dioclis, *quadraticem* Dinoſtrati, *logarithmicam* Naperi *curvam, catenariam, &c.*

Carteſius & Baker *conſtruxere æquationes trium & quatuor dimenſionum circuli & parabolæ ope:* Neutonus *eaſdem æquationes conſtruxit circuli & conchoidis* Nichomedis *ope:* Carteſius & Halleius *diverſas dedere methodos conſtruendi quinque & ſex dimenſionum æquationes:* Sluſius *protulit methodum ex ſubſtitutionibus inveniendi curvas, quarum ope datas æquationes algebraicas liceat conſtruere:* Cramer *vel* Euler *præmonet, ne in his ſubſtitutionibus curvas adhibeamus, quarum abſciſſæ fiunt impoſſibiles, cum ordinatæ fiant poſſibiles.* Walliſius *probavit cubicalem parabolam nullam diametrum habere.*

Carteſius *diſtinxit algebraicas curvas in diverſos ordines ſecundum dimenſiones æquationis relationem inter abſciſſam* (x) *& ejus correſpondentes ordinatas* (y) *exprimentis:* Barrow *nodos & interſectiones prædictarum curvarum, etiamque earum aſymptotos, inveniendi methodos docuit: E.g. Supponatur æquatio prædicta* $y = ax + b + \frac{c}{x} + \frac{d}{x^2} +$ &c. *erit æquatio ad aſymptotos* $v = ax + b$. *Ex.* 2. *Sit æquatio* $y^n + by^{n-1} +$ &c. $= x^n$ *&c. erit* $y = x - \frac{b}{n}$ &c. *aſſumpta recta linea, cujus æquatio eſt* $v = x - \frac{b}{n}$, *erit aſymptos datæ curvæ: hæc ſunt noſtri primi præceptoris in profeſſorio munere exempla, quæ ſatis manifeſto oſtendunt, illum quædam de doctrinâ infinitarum ſerierum literalium quantitatum excogitaſſe.*

Neutonus *dedit tres ſubſequentes algebraicarum curvarum proprietates;*

1. *Du-*

1. *Ducantur duæ rectæ inter ſe parallelæ curvam ſecantes in tot punctis, quot ſunt ejus dimenſiones* (n); *recta, quæ ita ſecat has parallelas, ut ſumma partium ex uno ſecantis latere ad curvam terminatarum æquetur partibus reliquis ex altero latere ad curvam terminatis, eodem modo ſecabit omnes alias hiſce parallelas, curvæque in* n *punctis occurrentes.* 2. *Si curva tot habeat aſymptotos, quot ſunt ejus dimenſiones, tum ſummæ partium ordinatæ inter ſingulum crus & ejus aſymptoton interceptarum ex utrâque parte diametri erunt inter ſe æquales.* 3. *Si curva tot habeat aſymptotos, quot ſunt ejus dimenſiones; tum contentum ſub omnibus abſciſſis inter ordinatam & abſciſſarum vertices interceptis erit in datâ ratione ad contentum ſub omnibus ordinatis: enumeravit etiam lineas tertii ordinis: invenit earum tres vel unicum axem, ſi qui ſint: dixit curvas* n *dimenſionum projicere umbras, quarum ſectiones ſunt curvæ* n *dimenſionum: transformavit geometrice datas curvas in alias eaſdem dimenſiones habentes; idem algebraice magis generaliter in hoc opere & Medit. Algebr. peragitur: ſubſequentes etiam proprietates conicarum ſectionum adjecit.* 1. *ſi rectæ duæ mobiles* BM & CM *per data puncta* B & C, *ceu polos ductæ, concurſu ſuo* M *deſcribant tertiam poſitione datam rectam* MN; *& aliæ duæ rectæ* BD, CD *cum prioribus duabus ad puncta illa data* B, C *datos angulos* MBD, MCD *efficientes ducantur; tum hæ duæ* BD, CD *concurſu ſuo* D *deſcribent conicam ſectionem per puncta* B, C *tranſeuntem.* 2. *Sit trapezium* ABCD *circa conicam ſectionem deſcriptum, cujus diagonales ſint* AC & BD; *biſecentur diagonales* AC & BD *in punctis* P & Q *reſpective; recta (per puncta* P & Q *ducta,) per centrum conicæ ſectionis tranſit.* 3. *Si rectæ tres tangant quamcunque coniſectionem, quarum duæ parallelæ ſint ac dentur poſitione; tum ſectionis ſemidiameter hiſce duabus parallela, erit media proportionalis inter harum ſegmenta, punctis contactuum & tangenti tertiæ interjecta; &c. ex his proprietatibus deſcripſit conicam ſectionem, quæ per data quinque puncta tranſibit; vel per data quatuor puncta tranſibit & rectam continget poſitione datam; &c. vel quæ rectas quinque poſitione datas continget.* 4. *Eſto ellipſeos umbilicus* S; *agatur* SP *ab umbilico* S *ad punctum ellipſeos* P, *ſecet diametrum* DK *lineæ tangenti ellipſim in puncto* P *parallelam in* E; *erit* EP *æqualis ſemiaxi majori.*

Coteſius *invenit ſubſequentem algebraicarum curvarum proprietatem; circa*

circa datum punctum A *revolvatur recta* AM, *quæ occurrat curvæ in tot punctis* M, M′, M″, *&c. quot ſunt ejus dimenſiones; & ſi in eâdem rectâ ſemper ſumatur* Pn, *ita ut* $\frac{1}{Pn} = \frac{1}{PM} + \frac{1}{PM'} + \frac{1}{PM''} +$ &c. *tum erit locus punctorum* n *recta: cui adjeci ſubſequentem propoſitionem; ſit* $\frac{a}{Pn^{2r+1}} + \frac{b}{Pn^{2s+1}} +$ &c. $= \frac{1}{PM^{2m+1}} + \frac{1}{PM'^{2m+1}} +$ &c. *ubi* r, s, *&c. integros numeros, qui ſunt vel æquales vel minores quam* m, *reſpective denotent; locus punctorum* n *curva erit haud majorum quam* 2 m + 1 *dimenſionum; &c.* Mac Laurin, *fere ex eodem principio, adjecit ſubſequentem algebraicarum curvarum proprietatem; occurrat recta quævis per punctum datum* P *ducta curvæ algebraicæ in tot punctis* (A, B, C, &c.) *quot ſunt ejus dimenſiones; rectæ* (AK, BL, CM, &c.) *curvam tangentes in his punctis ſecent abſciſſam, i.e. alteram rectam lineam* (PLM) *per idem datum punctum ductam in punctis* (K, L, M, &c.); *erit ſumma reciprocarum ſubtangentium* $\frac{1}{PK} + \frac{1}{PL} + \frac{1}{PM} +$ &c. *data quantitas:* Bernoulli *dedit ſequentis problematis ſolutionem; ſint* α & β *duæ curvæ ordinatæ, invenire curvas, in quibus* $\alpha^n + \beta^n$ *ſit data quantitas:* Cramer *vel* Euler *reddidit generalem methodum aſymptotos & diametros vel potius axes inveniendi; transformavit æquationem relationem inter abſciſſam & ejus correſpondentes ordinatas exprimentem, in æquationem relationem inter novam abſciſſam & ejus ordinatas deſignantem; docuit evaneſcere terminos nullius, unius, &c. dimenſionum, cum abſciſſa incipiat a puncto conjugato, vel puncto duplici, triplici, &c.* Clairaut *dedit æquationes ad curvas duplicis curvaturæ, & methodum earum tangentes ducendi docuit:* Euler *dedit æquationes ad algebraica ſolida (quas etiam dedi, priuſquam eas vel a* Clairaut *vel ab* Eulero *datas cognoviſſem); diviſit curvas ſecundi & tertii ordinis in* 16 & 138 *ſpecies; & ſolida ſecundi ordinis in quinque ſpecies; & eorum diametrialia plana inveſtigavit:* Lovatt *anglus propoſuit hanc propoſitionem, nempe, omnem rectilineam figuram, cujus ſingula latera dantur, & cujus area maxima ſit, in circulo inſcribi*

ſcribi poſſe; & ex iiſdem principiis ſequitur, ſi omnia latera præter unum dentur, in ſemicirculo inſcribi poſſe, cujus diameter eſt latus incognitum.

Reſtat, ut de rebus a nobis excogitatis pauca ſubjiciamus. In primo capite quædam novæ proprietates algebraicarum curvarum traduntur: E. g. 1. *Docetur methodus inveniendi, annon data abſciſſa ſit diameter; etiamque quot diametri eſſe poſſint in curvâ* n *dimenſionum, quæ ſecant ſuas ordinatas ad datum angulum.* 2. *Curvam, cujus omnes diametri ſint parallelæ, non conſtare ex hyperbolicis cruribus, ni omnes eorum aſymptoti ſint inter ſe parallelæ; & non conſtare ex parabolicis cruribus, ni omnia crura ſint concava vel convexa eâdem directione.* 3. *Dantur etiam formulæ æquationum, quando omnes diametri ſint parallelæ, vel ſeſe interſecent in uno eodemque puncto, vel unica ſolummodo detur generalis diameter.* 4. *Oſtenditur non poſſe plures eſſe quam* $\frac{n}{m}$ *inclinationes parallelarum ordinatarum, quæ ſecant curvam tantummodo in* n — m *punctis.* 5. *Probatur eſſe curvas imparium* (n) *dimenſionum, quæ ſecare poſſint rectam tantummodo in* 3, 5, 7, 9, *&c. punctis; parium* (n) *vero, quæ ſecant rectam tantummodo in* 2, 4, 6, *&c. punctis.* 6. *Deducuntur minimæ dimenſiones unius curvæ, quæ duci poſſit per data puncta duplicia, triplicia, &c. alterius curvæ.* 7. *Transformantur æquationes, relationes inter abſciſſas & earum correſpondentes ordinatas exprimentes, in æquationes relationes exprimentes inter lineas a duobus vel pluribus datis punctis ductas; ex æquationibus reſultantibus deduci poſſint proprietates prædictis analogæ.* 8. *Gyretur recta linea circa datum punctum, curvam* n *dimenſionum ſecans in* n *punctis; oſtenditur locum punctorum, quæ ita dividant has rectas, ut ſummæ ſegmentorum ex utrâque parte* ~~*abſciſſæ*~~ *ſint inter ſe æquales, haud eſſe curvam plurium quam* n *dimenſionum.* 9. *Datur proprietas algebraicarum curvarum ex ejus dimenſionibus deducenda.* 10. *Traduntur nonnullæ proprietates e terminis dimenſionum* n, n — 1, n — 2, *&c. in æquatione dimenſionum* n *pendentes: e. g. Loci centrorum curvæ dimenſionum* n, *& ultimarum diametrorum interſectionum erunt eædem curvæ ac loci centrorum & ultimarum diametrorum interſectionum ejus* (n) *aſymptotorum:* 2. *ducantur duæ rectæ curvam & ejus* n *aſymptotos reſpective ſecantes*

cantes in n *punctis; contentum ſub* n *abſciſſis erit ad contentum ſub earum* n *correſpondentibus ordinatis in eâdem ratione in curvâ ac in ejus aſymptotis.* 11. *Dantur quædam novæ obſervationes de conſtructione æquationum: e. g. de numero incognitarum quantitatum ex transformationibus abſciſſarum & ordinatarum duarum curvarum, quarum ope conſtruuntur datæ æquationes reſultantium; de evitandis impoſſibilibus radicibus in conſtructione æquationum; de caſibus in quibus curvæ per* $n \times m$ *vel* $n \times \frac{n+3}{2}$ *puncta ductæ, ubi puncta* $n \times m$ & $n.\frac{n+3}{2}$ *prædicta in curvis dimenſionum* (m & n) *reſpective ponantur, evadunt eædem; de caſibus in quibus æquatio ope curvarum dimenſionum* (n & m) *conſtructa haud aſcendet ad dimenſiones* ($n \times m$); *de reducendis ex conſtructione curvarum duabus æquationibus, in quibus continentur duæ incognitæ quantitates* x & y, *in unam, ita ut exterminetur altera incognita quantitas; ex earundem conſtructione de inveniendis impoſſibilibus datæ æquationis radicibus; de deſcribendis curvis, quæ tranſeant per puncta duplicia, &c.* 12. *Datâ curvâ algebraicâ* n *dimenſionum & ad datum punctum abſciſſæ datis* n *correſpondentibus ſubnormalibus, erit ſumma* n *ſubnormalium proportionalis diſtantiæ prædicti abſciſſæ puncti a puncto quodam immobili.* 13. *Dantur principia, e quibus algebraicè demonſtrari poteſt nullam algebraicam curvam, quæ continet in ſe ovalem ſeſe haud interſecantem, generaliter quadrari poſſe.* 14. *Datis duabus abſciſſis algebraicæ curvæ probatur ſemper inveniri poſſe ſummam arearum poſſibilium & impoſſibilium inter prædictas duas abſciſſas & earum ordinatas contentarum, prædictarum abſciſſarum & ordinatarum & earum functionum algebraicarum, & earundem circularium arcuum & logarithmorum, ope: idem mutatis mutandis affirmari poteſt de ſolidis e rotatione algebraicæ curvæ circa axem ſuum generatis; &c. His vero & conſimilibus adjiciuntur ſubſequentes propoſitiones:* 1. *docetur methodus, e datâ algebraicâ æquatione relationem inter abſciſſam & ejus correſpondentes ordinatas exprimente; & fluxionali quantitate, quæ eſt functio algebraica abſciſſæ* x, *ejus ordinatæ* y *& earum fluxionum, ſummam e ſingulis valoribus functionis prædictæ inveniendi:* 2.

quædam

quædam propoſitioni (nempe, ſi a datæ conicæ ſectionis puncto ad trapezii alicujus in conicâ illâ ſectione latera quatuor infinite producta, totidem rectæ in datis angulis ducantur, ſingulæ ad ſingulas; rectangulum ductarum ad oppoſita duo latera fore ad rectangulum ductarum ad alia duo latera in datâ ratione) adjiciuntur analogæ, quæ ad curvas ſecundi & omnes cujuſcunque ordinis applicantur. e. g. ſint tres rectæ, etiamque tres aliæ rectæ, quæ priores in novem punctis interſecant; contentum ſub lineis ductis a quibuſcunque punctis in curvâ ſecundi ordinis per novem puncta prædicta tranſeunte in datis angulis ad tres priores rectas ſemper erit in datâ ratione ad contentum ſub tribus lineis ductis ad tres poſteriores rectas in datis angulis ab iiſdem punctis. 3. datur proprietas quædam maxime elegans omnium parabolarum Archimedis conicæ parabolæ proprietati haud diſſimilis, & iſtâ proprietate generalior; quippe quæ ad omnes ſectiones abſciſſæ vel ad duas vel ad tres, quinque, &c. æquales partes extenditur. 4. adjicitur proprietas elegans parabolarum e Medit. Algeb. deducta; e. g. ſit æquatio $y = ax^{n} + bx^{n-1} + cx^{n-2} + \&c.$ ubi x ſit abſciſſa, y ejus ordinata & n integer numerus; erit æquatio ad abſciſſas, quarum ordinatæ ſint maximæ $nax^{n-1} + \overline{n-1}\,bx^{n-2} + \&c. = 0$; ſint $(n-1)$ ordinatæ maximæ poſſibiles, & ſecet data abſciſſa curvam in n punctis; tum erit $Pp \times Qq \times Rr \times Ss \times \&c. : ab^{2} \times ac^{2} \times bc^{2} \times ad^{2} \times bd^{2} \times cd^{2} \times \&c. :: a^{n-1} : n^{n}$ (Fig. 70.) 5. e propoſitione ſubſequente a me excogitatâ deducuntur plures proprietates algebraicarum curvarum; nempe, ſi ſint duæ æquationes relationes exprimentes inter abſciſſas & earum correſpondentes ordinatas ad duas curvas n & m dimenſionum reſpective, & reperiantur termini n, $n-1$, $n-2$, &c. m, $m-1$, $m-2$, &c. dimenſionum in duabus prioribus æquationibus iidem ac in duabus poſterioribus; tum omnem proprietatem duarum priorum curvarum, quæ ex iis terminis pendet, fore etiam proprietatem duarum poſteriorum, e. g. ſint duæ curvæ n dimenſionum, quarum termini n & $n-1$ dimenſionum in utrâque ſint iidem; ducantur ab eodem puncto in utrâque n^{2} rectæ perpendiculares ad utraſque curvas; tum ſummæ ordinatarum & abſciſſarum ad prædicta n^{2} puncta ambarum curvarum, quibus perpendiculares incidant, erunt inter ſe reſpective æquales: ultimo docetur methodus inveniendi

inveniendi algebraicam curvam, quæ habet quaſcunque datas lineas eam tangentes.

Hic forſan haud indignum eſt obſervatu multas ex his proprietatibus me haud deduxiſſe ex principiis algebræ vulgariter notis; at principia ipſa algebraica inveniſſe, ex quibus deductæ fuerint.

In capite ſecundo docentur methodi rectificandi curvoides vel epicurvoides; i. e. curvas generatas e rotatione datarum algebraicarum curvarum ſuper rectas vel ſuper alias algebraicas curvas; earum areas, radios curvaturæ, involutas, &c. inveniendi, in quibus traditur regula pro inveniendis tangentibus, radiis curvaturæ, &c. curvæ, a corpore duabus vel pluribus viribus agitato, deſcriptæ; invenitur etiam curva, quam deſcribit datum punctum in quâcunque datâ curvâ poſitum, cujus peripheria ſemper per datum punctum tranſeat & datam lineam in eo puncto tangat: aſſeritur omnes fluxionales curvas, i. e. curvas, quarum relationes inter abſciſſas & earum correſpondentes ordinatas exprimuntur per fluxionalem æquationem; deſcribi poſſe curvoidum vel epicurvoidum, ope.

In capite tertio continetur methodus inveniendi ſectionem ſolidi a rotatione cujuſcunque algebraicæ curvæ circa axem generati; etiamque quædam novæ proprietates ſolidorum e rotatione conicarum ſectionum circa axes ſuos generatorum; deinde traduntur quædam proprietates algebraicorum ſolidorum proprietatibus algebraicarum curvarum prius traditis analogæ, etiamque methodus eruendi projectionem plani vel ſolidi in planum vel ſolidum.

In capite quarto probatur ſingula latera polygoni circa ovalem in ſeſe ſemper concavam deſcripti ad puncta contactuum in æquales partes diviſa eſſe, cum polygonum ſit minimum ejuſdem numeri laterum, quod circa ovalem prædictam deſcribi poſſit. 2. *Sit polygonum triangulum vel trapezium, & ſi unicum ſolummodo triangulum vel trapezium hujuſce generis in datâ ovali inſcribi poſſit, tum minimum triangulum vel trapezium quod circa ovalem deſcribi poteſt reſpectivè erit quadruplum vel duplum maximi trianguli vel trapezii, quod in datâ ovali inſcribi poteſt: Si vero per hunc modum polygona ad omne punctum curvæ deſcribi poſſint, tum erunt omnia hæc polygona ſimiliter deſcripta inter ſe æqualia: Et ſic de pluribus conſimilibus hujuſce generis*

maximis

maximis & minimis; e. g. de maximis polygonis inſcriptis; de maximis inſcriptorum polygonorum, de minimis vero circumſcriptorum, laterum ſummis; &c.

Sequuntur quædam novæ & haud inelegantes conicarum ſectionum proprietates.

1. *Sectio ſphæroidis,* &c. *a quocunque plano per ejus focum tranſeunte facta habet focum prædictum pro foco ſuo.* 2. *Sectiones hyperboloidis & coni generati ab ejus aſymptotis circa axem ſuum rotantibus; & ſic ſectiones paraboloidis & cylindri, cujus axis eſt axis paraboloidis, ab eodem plano factæ, erunt ſimiles curvæ.* 3. *Sint duæ ſectiones duorum ſolidorum ſimiles; ex datis æquationibus ad duas conicas ſectiones, ex quarum rotationibus generantur ſolida, datur æquatio exprimens relationem inter ſinus angulorum, quos cum earum axe faciunt prædictæ ſectiones.* 4. *Parallellogrammum circa circulum deſcriptum erit vel quadratum vel rhombus, & cujuſcunque parallellogrammi circa conicam ſectionem deſcripti diagonales erunt conjugatæ diametri.* 5. *Datur regula pro detegendâ areâ cujuſcunque polygoni in parabolâ inſcripti.* 6. *Fig.* ρ. *Sit conica ſectio* $A\,\alpha\,\beta$, &c. E, *erit* $AB \times \alpha C \times \beta D \times$ &c. $\times EP = AP \times \alpha B \times \beta C \times$ &c. $\times ED$. 7. *Fig.* π. *Sint* $AB = d$, $CD = c$, & *ſemidiameter conicæ ſectionis lineis* A B & C D *parallela* $= t$; *tum erunt* $A\alpha = \frac{d}{2} \pm \sqrt{\frac{d^2 c - 4 t^2 d}{4 c}}$, &c. 8. *Datur methodus inſcribendi trapezium in datâ ellipſi dato trapezio ſimile, & circa eam circumſcribendi parallellogrammum dato parrallellogrammo ſimile, ſi modo inſcribi vel circumſcribi poſſint.* 9. *Fig. C. Sit ellipſis* P Q R S, &c. *deſcribatur circa eam polygonum* p q r s, &c. *erit contentum* $Pp \times Qq \times Rr \times$ &c. $= Pq \times Qr \times Rs \times$ &c. 10. *Deſcribantur duo polygona* n *laterum circa eandem ellipſim, ita ut eorum latera ad puncta contactuum iiſdem rationibus dividantur, erunt hæc duo polygona inter ſe æqualia.* 11. *Jungantur puncta vel contactuum vel angulorum prædictorum polygonorum & ellipſeos centrum, aggregatum quadratorum*

rum ex ſingulis hiſce lineis in utriſque polygonis idem reſpective evadet. 12. *Inſcribantur in eâdem ellipſi duo polygona* n *laterum, ita ut duæ proximæ lineæ æquales angulos cum tangente ad idem punctum contactûs utrinque faciant, erunt ſummæ laterum amborum polygonorum reſpective inter ſe æquales.* 13. *Fig.* D. *Sint duo polygona* n *laterum* A B C D E, *&c.* & P Q R S T, *&c. Si linea* A C *parallela ſit lineæ tangenti conicam ſectionem in* B, *linea* B D *parallela tangenti in* C, *&c, & ſic linea* P R *tangenti in* Q, *linea* Q S *tangenti in* R, *&c. tum erunt duo polygona* A B C D, *&c.* & P Q R S, *&c. inter ſe æqualia.* 14. *Si omne latus dividatur in reciprocâ ratione ſummæ vel differentiæ ſecantium & tangentium complementi angulorum polygoni lateris ſegmentis adjacentium, erunt ſummæ laterum duorum polygonorum inter ſe æquales.* 15. *Si circa conjugatas diametros ſolidi a rotatione ellipſeos circa axem ſuum generati deſcribantur elliptici cylindri, erunt hi cylindri inter ſe æquales.* 16. *Idem etiam affirmari poteſt de ſolidis circa prædicta generata ſolida deſcriptis, quorum latera rectilinea perpetuo in eâdem ratione ad puncta contactuum dividuntur, erunt enim omnia hæc ſolida ſibi invicem æqualia.* 17. *Minimi circumſcripti & maximi inſcripti coni ad omne punctum in prædictis ſolidis erunt ſibi invicem æquales.* 18. *Si conica ſectio gyretur circa axem ſuum, & a dato puncto ducantur tangentes, uſque donec redeant iterum ad idem punctum; tum erunt duo contenta ſub ſingulis lineis ~~inter~~ puncta, in quibus duæ lineæ ſucceſſivæ ſibi invicem occurrant, & puncta contactuum rectarum & conicæ ſectionis, alternatim ſumptis, inter ſe æqualia.* 19. *Si quatuor rectæ a dato puncto ducantur perpendiculares ad hyperbolam in quatuor punctis* α, β, γ & δ; *& duæ rectæ ab eodem puncto ad ejus aſymptotos in punctis* π & ρ, *ducantur perpendiculares; tum, ſi ducantur ordinatæ a punctis* α, β, γ & δ *ad quamlibet hyperbolæ abſciſſam, quibus ducantur duæ parallelæ a punctis* π & ρ *ad eandem abſciſſam; erit dupla ſumma harum duarum linearum æqualis ſummæ quatuor ordinatarum prædictarum. Omnes hæ propoſitiones mutatis mutandis æque de hyperbolis, & hyperboloidibus ac de ellipſibus & ſphæroidibus affirmari poſſunt.*

Non

Non hìc enumerantur lineæ quarti ordinis, quoniam talis enumeratio nec in detegendis curvarum proprietatibus, nec in quâvis aliâ re ulli uſui inſervire poteſt; etiamque in enumeratione linearum ſuperiorum ordinum ſecundum methodos a Newtono *vel* Eulero *uſitatas occurrunt aliæ curvæ, quæ quadraturam recipiunt; aliæ vero curvæ ejuſdem ſpeciei, quæ minime quadrari poſſunt; ob has & alias rationes omnes enumerationes hujuſce generis hìc rejiciuntur.*

ALGEBRAICARUM

CURVARUM PROPRIETATES.

CAPUT I.

DEF. 1. Sit data æquatio n dimenſionum relationem inter abſciſſam & ejus correſpondentes ordinatas definiens $Ay^n + \overline{a+bx}\, y^{n-1} + \overline{c+dx+ex^2}\, y^{n-2} +$ &c. $= o$, & ſi, abſciſâ eâdem manente, ita mutari poſſit ordinatarum ad abſciſſam inclinatio, ut æquatio relationem inter prædictam abſciſſam (z) et ejus correſpondentes ordinatas (v) deſignans, ſit hujuſce formulæ $v^n * + \overline{C+Dz+Ez^2}\, v^{n-2} +$ &c. $= o$, ubi terminus $\overline{A+Bz}\, v^{n-1}$ evaneſcat, tum ea abſciſſa dici poſſit diameter primi generis.

2. Datâ prædictâ æquatione $Ay^n + \overline{a+bx}\, y^{n-1} +$ &c. $= o$, & ſi abſciſâ eâdem manente, ita transformari poſſit ordinatarum inclinatio, ut reſultans æquatio relationem inter prædictam abſciſſam (z) & ejus correſpondentes ordinatas (v) ſit hujuſce formulæ, $* \overline{A+Bz}\, v^{n-1} * + \overline{F+Gz+Hz^2+Kz^3} \times v^{n-3} +$ &c. $= o$, ubi evaneſcant termini $v^n + \overline{C+Dz+Ez^2}\, v^{n-2}$ tum abſciſſa dici poteſt diameter ſecundi generis; ſi vero reſultans æquatio ſit hujuſce formulæ $** \overline{C+Dz+Ez^2}\, v^{n-2} * + \overline{L+Mz+Nz^2+Oz^3+Pz^4}\, v^{n-4} +$ &c. $= o$, ubi evaneſcant termini v^n, $\overline{A+Bz}\, v^{n-1}$ & $\overline{F+Gz+Hz^2+Kz^3}\, v^{n-3}$ tum abſciſſa dici poteſt diameter tertii generis, & ſic deinceps. Si ordinatæ (v) ſecant curvam in (n) punctis, tum ſumma ordinatarum ex unâ parte diametri primi generis æqualis erit ſummæ ordinatarum ex altera.

Si vero ordinatæ (v) ad diametrum ſecundi generis ſecant curvam in ($n-1$) punctis, tum ſumma ordinatarum ex una parte diametri æqualis erit ſummæ ordinatarum ex alterâ, & ſic deinceps.

Hoc conſtat e vulgaris algebræ principiis.

3. Diametri vel potius axes nonnunquam definiuntur eſſe lineæ, quæ ita ſecant ordinatas ſibi ipſis ad rectum angulum inſiſtentes, ut ſingula ordinata ex unâ parte diametri alteram habeat ex oppoſitâ parte ſibi ipſi æqualem.

4. Lineæ geometricæ in ordines ſecundum numerum dimenſionum æquationis, quâ relatio inter ordinatas & abſciſſas definitur, uti bene monet Carteſius, & non ſecundum numerum punctorum, in quibus a rectâ lineâ ſecari poſſint, diſtingui debent: pars enim poſterior Neutonianæ definitionis aliquando fallit.

5. Per dimenſiones curvæ, intelligo dimenſiones æquationis relationem inter abſciſſam & ejus ordinatas exprimentis.

PROB. I.

Fig. 1. Datâ algebraicâ æquatione relationem inter abſciſſam ($AP=x$) & ejus ordinatas ($PM=y$) definiente, invenire æquationem relationem definientem inter abſciſſam (ap), quæ ſecat abſciſſam (AP) ad datum angulum (pNP) & in dato puncto (N), & ordinatas (pM, &c.) eam (ap) ſecantes in dato angulo (MpN).

Sit radius (1), & ſinus angulorum APM, PNp, & MpN reſpective s, S & σ, tum per trigonometriam ſinus angulorum NrP & rMp erunt $s\sqrt{1-S^2}+S\sqrt{1-s^2}=c$ & $\sigma\sqrt{1-c^2}+c\sqrt{1-\sigma^2}=C$; ſubſtituantur pro lineis NA, Na, ap, pM reſpective a, b, z, v; & erit $z+b-\frac{C}{c}v=Nr$, & $\frac{cz+cb-Cv}{s}+a=x$, & $y=rM+rP=\frac{\sigma v}{c}+\frac{Sz+Sb}{s}-\frac{CS}{cs}v=\frac{\sigma s-CS}{cs}v+\frac{Sz}{s}+\frac{Sb}{s}$; quibus quantitatibus pro ſuis valoribus (x & y) in datâ æquatione ſubſtitutis, reſultat

ſultat æquatio relationem inter abſciſſam (ap), & ejus ordinatas (pM) definiens.

Fig. 2. Cor. 1. Datâ æquatione relationem inter abſciſſam ($AP=x$) & ejus ordinatas ($PM=y$) definiente, invenire æquationem inter abſciſſam ($Ap=z$) & ejus ordinatas ($pM=v$) relationem exprimentem.

Fig. 1. In hoc caſu NA (a), Na (b) & angulus pNP nihilo evadunt æquales, & conſequenter ſin. $\angle APM = s = c =$ ſin. $\angle NrP$, & ſin. $\angle rMp = C = \sigma\sqrt{1-s^2} + s\sqrt{1-\sigma^2} =$ ſin. $\angle PMp$ (Fig. 2); & exinde $x = \frac{cz + cb - Cv}{s} + a = z + \frac{s\sqrt{1-\sigma^2} + \sigma\sqrt{1-s^2}}{s} \times v$ & $y = \frac{\sigma s - CS}{cs}v + \frac{Sz}{s} + \frac{Sb}{s} = \frac{\sigma}{s}v$, quibus valoribus pro x & y in datâ æquatione ſubſtitutis, reſultat æquatio quæſita. Aliter: Fig. 2.

Sint ſinus angulorum APM & ApM reſpective s & σ, & ſinus anguli (pMP) erit $s\sqrt{1-\sigma^2} + \sigma\sqrt{1-s^2}$; pro ordinatâ PM (y) in datâ æquatione ſubſtituatur ejus valor $\frac{\sigma v}{s}$, & pro abſciſsâ AP (x) ſubſtituatur $z + \frac{s\sqrt{1-\sigma^2} + \sigma\sqrt{1-s^2}}{s} \times v$, & conſit corollarium.

Fig. 3. Cor. 2. Datâ prædictâ æquatione relationem inter $AP=x$ & $PM=y$ exprimente, invenire æquationem relationem inter abſciſſam Np vel Nr (z) & ordinatam Mp (v) exprimentem.

Fig. 1. In hoc caſu Na (b) $= o$, & $\sigma =$ ſin. $\angle MpN =$ ſin. $\angle MrN =$ ſin. $\angle NrP = s\sqrt{1-S^2} + S\sqrt{1-s^2} = c$, & angulus rMp & ejus ſinus C nihilo æqualis erit; unde $x = \frac{cz + cb - Cv}{s} + a = \frac{s\sqrt{1-S^2} + S\sqrt{1-s^2}}{s}z + AN$ (a), & $y = \frac{\sigma s - CS}{cs}v + \frac{Sz}{s} + \frac{Sb}{s} = v + \frac{Sz}{s}$, quibus quantitatibus in datâ æquatione pro ſuis valoribus (x & y) ſubſtitutis, reſultat æquatio quæſita. Aliter: Fig. 3.

Sint ſinus angulorum APM & PNp reſpective s & S, & ſinus an-

guli (MpN) erit $s\sqrt{1-S^2}+S\sqrt{1-s^2}$; pro abſciſsâ $AP\ (x)$ in datâ æquatione ſcribatur ejus valor $\frac{s\sqrt{1-S^2}+S\sqrt{1-s^2}}{s}\,z+AN\,(a)$; & pro ordinatâ y, $v+\frac{Sz}{s}$; & reſultat æquatio quæſita.

Fig. 1. Iiſdem literis eaſdem quantitates ac in problemate denotantibus, & datâ æquatione relationem inter abſciſſam $AP\ (x)$ & ejus ordinatas $PM\ (y)$ & earum incrementa ($\dot{x}$, $\ddot{x}$, $\dddot{x}$, &c. $\dot{y}$, $\ddot{y}$, $\dddot{y}$, &c.) exprimente, hinc inveniri poſſit æquatio relationem inter abſciſſam (ap) & ordinatas (pM) & earum incrementa denotans.

Scribantur pro x, $\dot{x}$, $\ddot{x}$, &c. reſpective $\frac{cz+cb-Cv}{s}+a$, $\frac{c\dot{z}-C\dot{v}}{s}$, $\frac{c\ddot{z}-C\ddot{v}}{s}$, &c. & pro y, $\dot{y}$, $\ddot{y}$, &c. ſcribantur reſpective $\frac{\sigma s-CS}{cs}\,v+\frac{Sz}{s}+\frac{Sb}{s}$, $\frac{\sigma s-CS}{cs}\,\dot{v}+\frac{S\dot{z}}{s}$, $\frac{\sigma s-CS}{cs}\,\ddot{v}+\frac{S\ddot{z}}{s}$, &c. & reſultat æquatio quæſita.

PROB. II.

Fig. 2. Datâ æquatione n dimenſionum $Ay^n+\overline{a+bx}\,y^{n-1}+\overline{c+dx+ex^2}\,y^{n-2}+\overline{f+gx+hx^2+kx^3}\,y^{n-3}+$ &c. $=0$, relationem inter abſciſſam $AP=x$ & ejus ordinatas $PM=y$ exprimente; ubi Ay^n, bxy^{n-1}, ex^2y^{n-2}, kx^3y^{n-3}, &c. ſint termini n dimenſionum; & ay^{n-1}, dxy^{n-2}, hx^2y^{n-3}, &c. ſint termini $n-1$ dimenſionum; invenire utrum abſciſſa AP ſit diameter primi generis, necne.

Sint s & σ reſpective ſinus dati anguli APM & quæſiti ApM, quem veræ ordinatæ $pM\ (v)$ (i. e. ordinatæ, quarum ſummæ ex utroque diametri latere ſint inter ſe æquales) cum ſuâ diametro faciunt: & per Cor. 1. Prob. 1. pro y ſcribatur $\frac{\sigma v}{s}$, & pro x ſcribatur z (abſciſſa ad veras ordinatas,) $+\frac{s\sqrt{1-\sigma^2}+\sigma\sqrt{1-s^2}\ (C)}{s}\,v$, & reſultat æquatio

A

$$\overline{A\frac{\sigma^n}{s_n} + b\frac{\sigma^{n-1}}{s^{n-1}}C + e\frac{\sigma^{n-2}}{s^{n-2}}C^2 + k\frac{\sigma^{n-3}}{s^{n-3}}C^3 + \&c.} \times v^n + \overline{b\frac{\sigma^{n-1}}{s^{n-1}} + 2e\frac{\sigma^{n-2}}{s^{n-2}}C + 3k\frac{\sigma^{n-3}}{s^{n-3}}C^2 + \&c.} \times z v^{n-1} + a \times \overline{\frac{\sigma^{n-1}}{s^{n-1}} + d\frac{\sigma^{n-2}}{s^{n-2}}C + b\frac{\sigma^{n-3}}{s^{n-3}}C^2 + \&c.} \times v^{n-1} + \&c. = o.$$

Ut fiat abſciſſa diameter primi generis, neceſſe eſt ut evaneſcant termini $z v^{n-1}$ & v^{n-1}, & conſequenter eorum coefficientes $b\frac{\sigma^{n-1}}{s^{n-1}} + 2e\frac{\sigma^{n-2}}{s^{n-2}}C + 3k\frac{\sigma^{n-3}}{s^{n-3}}C^2 + \&c$, & $a\frac{\sigma^{n-1}}{s^{n-1}} + d\frac{\sigma^{n-2}}{s^{n-2}}C + b\frac{\sigma^{n-3}}{s^{n-3}}C^2 + \&c.$ nihilo evadant æquales, ſed haud evaneſcat terminus (v^n) i. e. ejus coefficiens $A\frac{\sigma^n}{s^n} + b\frac{\sigma^{n-1}}{s^{n-1}}C + e\frac{\sigma^{n-2}}{s^{n-2}}C^2 + \&c.$ haud nihilo æqualis fiat.

Fiant igitur coefficientes terminorum v^{n-1} & $z\,v^{n-1}$ nihilo æquales, & in duabus reſultantibus æquationibus pro $\frac{\sigma}{sC}$ ſubſtituatur q, & reſultant duæ æquationes $b q^{n-1} + 2 e q^{n-2} + 3 k q^{n-3} + \&c, = o$, & $a q^{n-1} + d q^{n-2} + b q^{n-3} + \&c, = o$; ſi igitur hæ duæ æquationes communem & poſſibilem habeant radicem, quæ haud ſit etiam radix æquationis $A q^n + b q^{n-1} + e q^{n-2} + k q^{n-3} + \&c. = o$, tum abſciſſa erit diameter primi generis.

Si nullus terminus datæ æquationis ſit (n) dimenſionum, niſi primus Ay^n i. e. evaneſcant termini bxy^{n-1}, ex^2y^{n-2}, &c. tum abſciſſa erit diameter primi generis, ſi modo æquatio $a q^{n-1} + d q^{n-2} + b q^{n-3} + \&c. = o$, poſſibilem habeat radicem: ſi vero nulli in datâ æquatione continentur termini $a y^{n-1}$, dxy^{n-2}, $bx^2 y^{n-3}$, &c, $(n-1)$ dimenſionum, tum abſciſſa erit diameter prædicti generis, ſi modo radix æquationis $b q^{n-1} + 2 e q^{n-2} + 3 k q^{n-3} + \&c. = o$, ſit poſſibilis, quæ etiam haud ſit radix æquationis $A q^n + b q^{n-1} + e q^{n-2} + \&c, = o$; vel ſi radices impoſſibiles pro radicibus eodem modo, quo poſſibiles habeantur, tum in his duobus caſibus abſciſſa erit primi generis diameter.

Cor. 1. Hinc diameter primi generis poſſit habere $(n-1)$ diverſas incli-

inclinationes verarum ordinatarum, & haud plures, ni infinitæ ſunt: duæ enim æquationes $bq^{n-1} + 2eq^{n-2} + 3kq^{n-3} +$ &c. $= 0$; $aq^{n-1} + dq^{n-2} + hq^{n-3} +$ &c, $= 0$; poſſunt habere $n-1$ diverſos communes diviſores & haud plures.

Cor. 2. Poſſunt eſſe n & haud plures diverſæ inclinationes ordinatarum ad abſciſſam, in quibus ordinatæ haud ſecant curvam in n punctis vel poſſibilibus vel impoſſibilibus, poſſunt enim eſſe (n) radices æquationis $Aq^n + bq^{n-1} + eq^{n-2} + kq^{n-3} +$ &c, $= 0$, & haud plures.

PROB. III.

Fig. 4. Datâ prædictâ æquatione $Ay^n + \overline{a+bx}\,y^{n-1} +$ &c, $= 0$, relationem inter abſciſſam NP (x) & ejus ordinatas PM (y) exprimente, invenire numerum diametrorum, quæ ſecare poſſunt ſuas ordinatas ad datum angulum NpM.

Sint ſinus datorum angulorum MpN & MPN reſpective S & s, & ſinus anguli quæſiti Mrp (c). Per Problema 1, transformetur data æquatio in æquationem relationem inter abſciſſam Np (z) & ejus ordinatas Mp (v) deſignantem, i. e. in datâ æquatione pro x ſcribatur ejus valor $\frac{cz}{s} - \frac{S\sqrt{1-c^2} + c\sqrt{1-S^2}}{s} \times v$, & pro y ejus valor $\frac{S}{c}v = \frac{c\sqrt{1-s^2}+s\sqrt{1-c^2}}{s} \times \frac{S\sqrt{1-c^2}+c\sqrt{1-S^2}}{c} \times v + \frac{c\sqrt{1-s^2}+s\sqrt{1-c^2}}{s} \times z$; pro $c\sqrt{1-s^2} + s\sqrt{1-c^2}$ & $S\sqrt{1-c^2} + c\sqrt{1-S^2}$ ſcribantur reſpective t & C, & reſultant $\frac{cz}{s} - \frac{Cv}{s} = x$ & $\frac{sS-Ct}{sc}v + \frac{t}{s}z = y$; & æquationis ex hac transformatione reſultantis termini (zv^{n-1}) coefficiens erit $n \times \left(\frac{sS-Ct}{cs}\right)^{n-1} \times \frac{t}{s}A + \overline{\left(\frac{sS-Ct}{cs}\right)^{n-1} \times \frac{c}{s} - \overline{n-1}\left(\frac{sS-Ct}{cs}\right)^{n-2} \times \frac{t}{s} \times \frac{C}{s}} \times b - \overline{\left(\frac{sS-Ct}{cs}\right)^{n-2} \times \frac{2Cc}{s^2} - \overline{n-2} \times \left(\frac{sS-Ct}{cs}\right)^{n-3} \times \frac{C^2}{s^2} \times \frac{t}{s}} \times e$ &c.

Pro

Pro literis t & C ſcribantur earum prædicti valores, & evadet $Ct = Ss - Ssc^2 + (S\sqrt{1-s^2} + s\sqrt{1-S^2})c\sqrt{1-c^2} + c^2\sqrt{1-s^2} \times \sqrt{1-S^2}$, unde

$$\frac{Ss-Ct}{cs} = \frac{Ssc - (S\sqrt{1-s^2}+s\sqrt{1-S^2})\sqrt{1-c^2} - c\sqrt{1-s^2}\times\sqrt{1-S^2}}{s},$$

ſubſtituatur hic valor pro $\frac{Ss - Ct}{cs}$ in prædictâ coefficiente, & fiat quantitas reſultans nihilo æqualis, in æquatione reſultante incognita quantitas c haud plures habet quam n dimenſiones, ita reducatur hæc æquatio ut exterminetur irrationalis quantitas $\sqrt{1-c^2}$, tum haud aſcendet quantitas c in æquatione reſultante ad plures quam $2n$ dimenſiones, & conſequenter haud habebit plures diverſos valores quam $2n$, quorum ſingulus affirmativus habet negativum ei æqualem, hi autem duo valores unum angulum Mrp ſolummodo præbent, ergo ſolummodo poſſint eſſe (n) diametri, quæ ad datum angulum ſuas ordinatas ſecant.

Si ordinata per centrum diametri tranſeuns ſit diameter ad ordinatas diametro prædicto parallelas, tum omnes ejuſmodi diametri prædictis ſunt adjungendæ.

THEOR. I.

Fig. 3. Æquationis exprimentis relationem inter abſciſſam ($x = AP$), & ejus correſpondentes ordinatas ($y = PM$) ſint termini, in quibus quantitatum (x & y) dimenſiones inveniuntur maximæ, $\overline{Ay+Bx}^n$; & impoſſibiles radices eodem modo ac poſſibiles habeantur; tum omnes curvæ diametri erunt inter ſe parallelæ, & vice verſâ omnis linea cuicunque diametro parallela (excepto caſu in theor. 4. dato) erit diameter.

Per Cor. 2. Prob. 1. ita transformetur prædicta æquatio, ut exprimat relationem inter abſciſſas Np (z) & ejus ordinatas pM (v), ubi $AN = o$ & abſciſſa Np facit angulum cum abſciſsâ NP, cujus ſinus ſit S.

Sit ſinus dati anguli NPM (s), ſinus anguli NpM erit $s\sqrt{1-S^2} + S\sqrt{1-s^2} = C$: Sit $C : -S :: 1 : \frac{B}{A}$, & exinde $S = \frac{Bs}{\sqrt{A^2+B^2+2AB\sqrt{1-s^2}}}$:

Pro

Pro abſciſsâ x & ordinatâ y in datâ æquatione ſcribantur earum reſpectivi valores $\frac{Cz}{s}$ & $v+\frac{Sz}{s}$, & conſequenter pro $y+\frac{Bx}{A}$ ejus valor v, reſultat æquatio in quâ terminus, in quo dimenſiones abſciſſæ (z) & ordinatæ v inveniuntur maximæ, erit v^n; i.e. æquatio reſultans erit hujuſce formulæ $v^n+\alpha v^{n-1}+\overline{\gamma+\delta z}\,v^{n-2}+$ &c. $=0$, & ſimiliter æquatio relationem inter quamcunque abſciſſam huic parallelam & ejus correſpondentes ordinatas deſignans erit ejuſdem formulæ.

Sin æquatio relationem inter abſciſſam (z) & ejus correſpondentes ordinatas (v) exprimens ſit hujuſce formulæ, tum abſciſſa erit diameter.

Cor. 1. Linea huic diametro haud parallela, non erit diameter; æquatio enim relationem inter abſciſſam & ejus correſpondentes ordinatas exprimens, non erit prædictæ formulæ, & conſequenter abſciſſa non erit diameter.

Cor. 2. In hoc caſu duæ & haud plures poſſunt eſſe diametri, quæ datum angulum cum ſuis ordinatis faciunt; ni rectus ſit angulus, tum unica ſolummodo erit diameter.

Cor. 3. Non conſtare poteſt curva n dimenſionum, cujus omnes diametri ſunt parallelæ, ex (n) hyperbolicis cruribus, ni omnes eorum aſymptoti ſint inter ſe parallelæ. Termini enim, in quibus maximæ inveniuntur datæ æquationis dimenſiones, erunt $\overline{y+\frac{Ax}{B}}^n$, ſi diametri ſint inter ſe parallelæ. Termini vero, in quibus maximæ inveniuntur dimenſiones iidem erunt in æquatione ad n aſymptotos ac ad datam curvam, ſed termini in quibus maximæ inveniuntur dimenſiones æquationis ad n aſymptotos haud poſſint eſſe $\overline{y+\frac{Bx}{A}}^n$, ni omnes ſint parallelæ; ergo omnes aſymptoti datæ curvæ erunt inter ſe parallelæ.

Cor. 4. Non conſtat prædicta curva ex parabolicis cruribus, ni omnia crura ad infinitam diſtantiam ſint concava vel convexa ad eaſdem partes.

E ſubſequentibus æquationibus ad hyperbolica & parabolica crura hæc facile conſtant.

Cor.

Cor. 5. Æquationis prædictæ $v^n + \alpha v^{n-1} + \overline{\gamma + \delta z}\, v^{n-2} +$ &c. $= o$, ſint pares dimenſiones (i. e. ſit n par numerus), & nulli termini $(n-1)$ dimenſionum in datâ æquatione contineantur, in quibus inveniuntur impares dimenſiones ordinatæ v; tum ad eandem diſtantiam ex utrâque parte diametri, quæ rectum angulum cum ſuis ordinatis facit, eadem erit ordinatarum ad diametrum inclinatio.

Hoc facile conſtabit e præcedentibus ſubſtitutionibus.

THEOR. II.

Fig. 3. Æquationis n dimenſionum relationem inter abſciſſam $AP = x$ & ejus correſpondentes ordinatas $PM = y$ exprimentis ſint nulli termini, in quibus inveniuntur $(n-1)$ dimenſiones, tum omnes curvæ diametri ſeſe interſecant in puncto, quod eſt abſciſſæ vertex.

Per Cor. 2. Prob. 1. Transformetur hæc æquatio in alteram, relationem inter abſciſſam Ap (z) ad datam abſciſſam AP (x) utcunque inclinatam & ejus correſpondentes ordinatas pM (v), exprimentem; ubi $AN = o$; tum ſemper reſultat æquatio hujuſce formulæ $v^n + bzv^{n-1} + \overline{c + ez^2} \times v^{n-2} +$ &c. $= o$, in quâ deſunt omnes termini $n-1$ dimenſionum, & per Prob. 2. abſciſſam hujus æquationis eſſe diametrum conſtat, & conſequenter omnes lineas per hoc centrum ductas eſſe diametros.

Cor. 1. Sit curva n dimenſionum n habens aſymptotos ſeſe ſecantes in uno eodemque puncto; tum iſtud punctum erit curvæ centrum.

Fig. 5. Cor. 2. Si generale centrum non eſt vertex abſciſſæ, ſubſtituantur pro abſciſsâ AP (x) ejus valor $(ap + \alpha = z + \alpha)$, & pro ordinatâ $PM(y)$ ejus valor $(pM + \beta = v + \beta)$, & ita aſſumantur α & β, ut nihilo fiant æquales termini, in quibus $(n-1)$ dimenſiones abſciſſæ (z) & ejus ordinatarum (v) inveniuntur; tum vertex novæ abſciſſæ erit centrum quæſitum.

Cor. 3. Si curva habet ~~parabolicum crus~~ omne crus possibile vel impossibile, parabolicum hujuſce formulæ $(y^{m+r} = ax^m)$, in quâ $(x$ & $y)$ ſunt abſciſſæ & ordinatæ reſpective, & r non eſt minor quam 2; tum neceſſario habet centrum generale.

Hoc enim in caſu evaneſcunt prædicti $n-1$ dimenſionum termini.

Diameter haud erit ordinata ad ſuam ordinatam per centrum ejus tranſeuntem, ni termini $a\,(o)\,y^{n-1}$, $b\,(o)\,xy^{n-1}$, $p\,(o)\,x^{n-1}y$ æquationis $y^n + \overline{a + bx}\,y^{n-1} + \&c. = o$, relationem inter abſciſſam diametri & ejus ordinatas exprimentis, nihilo ſunt reſpective æquales.

THEOR. III.

Fig. 2. Curva n dimenſionum non habere poteſt plures quam $(n-1)$ parallelas diametros, ni omnes ejus diametri ſint parallelæ.

Sit enim angulus APM rectus, & pro ordinatâ $PM\,(y)$ ſubſtituantur Sv (ubi $v = Mp$ & $S =$ ſin; anguli MpP), & pro ejus abſciſsâ $AP\,(x)$, $z\,(Ap) - \sqrt{1 - S^2}\,v$, in datâ æquatione $y^n + \overline{a + bx}\,y^{n-1} + \overline{c + dx + ex^2}\,y^{n-2} + \&c. = o$; tum reſultat termini (zv^{n-1}) coefficiens $bS^{n-1} - 2eS^{n-2}\sqrt{1 - S^2} + 3kS^{n-3} \times \overline{1 - S^2}$, &c.

Fiat hæc coefficiens nihilo æqualis, & ita transformetur reſultans æquatio, ut exterminentur irrationales quantitates; reſultat æquatio haud plurium quam $(2 \times \overline{n-1})$ dimenſionum, & conſequenter haud plures quam $(2 \times \overline{n-1})$ habebit radices, $(\overline{n-1})$ affirmativas, & $(\overline{n-1})$ negativas; negativæ & affirmativæ erunt reſpective inter ſe æquales, & conſequenter ad eandem ordinatarum inclinationem referentur negativus & ejus æqualis affirmativus valor; & numerus parallelarum diametrorum non major erit quam $(n-1)$, ni omnes ſint parallelæ.

Eodem modo ratiocinari licet de diametris in uno eodemque puncto ſeſe ſecantibus.

THEOR. IV.

Curva algebraica poteſt unicam ſolummodo habere diametrum, quam dico generalem diametrum, cui lineæ utcunque inclinatæ erunt ordinatæ.

Hujuſmodi vero diametrum habet curva, ſi æquationis relationem inter abſciſſam & ejus correſpondentes ordinatas exprimentis termini, in quibus dimenſiones inveniuntur maximæ, ſint $(\overline{Ay + Bx}^n)$, & nulli $(n-1)$ dimenſionum inveniantur termini.

Hoc

Hoc in caſu e prædictis conſtat curvam habere omnes ſuas diametros inter ſe parallelas, & omnes ejus diametros ſeſe ſecare in uno eodemque puncto, & conſequenter curvam unicam ſolummodo habere diametrum.

Transformetur enim æquatio relationem inter abſciſſam & ejus correſpondentes ordinatas exprimens in alteram, in quâ terminus, in quo inveniuntur maximæ dimenſiones, ſit ſolummodo (v^n), & nulli ſint ($n-1$) dimenſionum termini, tum abſciſſa erit generalis diameter.

Cor. Si curva habet ~~parabolicum crus~~ [omne crus possibile vel impossibile parabolicum] hujuſce generis ($y^{m+r} = ax^m$), in quâ r non eſt minor quam (3), tum generalem habet diametrum.

THEOR. V.

Fig. 2. Non poſſunt eſſe plures quam $\frac{n}{m}$ diverſæ inclinationes parallelarum ordinatarum, quæ ſolummodo ſecant curvam in $n-m$ punctis vel poſſibilibus vel impoſſibilibus.

Sit enim æquatio relationem deſignans inter abſciſſam ($AP = x$) & ejus correſpondentes ordinatas ($PM = y$) cum eâ rectum angulum facientes $y^n + \overline{a + bx}\, y^{n-1} + \overline{c + dx + ex^2}\, y^{n-2} + \overline{f + gx + hx^2 + kx^3}\, y^{n-3} + \&c. = 0$. Transformetur hæc æquatio, ita ut abſciſſæ eædem maneant, ordinatarum vero mutetur inclinatio: i. e. per Cor. 1. Prob. 1. pro ordinatâ y ſubſtituatur Sv; & pro abſciſsâ (x), $z + \sqrt{1 - S^2}\, v$, ubi S, z & v reſpective denotant ſinum anguli MpP, abſciſſam Ap, & ejus ordinatas pM; tum reſultat æquatio $\overline{S^n + bS^{n-1}\sqrt{1 - S^2} + eS^{n-2} \times \overline{1 - S^2} + kS^{n-3} \times \overline{1 - S^2}^{1\frac{1}{2}} + \&c.} \times v^n + \&c. = 0$: ut hæc abſciſſa fiat diameter vel abſciſſa ſecundi generis, neceſſe eſt, ut evaneſcat terminus v^n, & conſequenter ejus coefficiens nihilo fiat æqualis, i. e. $S^n + bS^{n-1}\sqrt{1 - S^2} + eS^{n-2} \times \overline{1 - S^2} + \&c. = 0$; quâ æquatione reductâ, haud aſcendit æquatio reſultans ad majores quam $2n$ dimenſiones, cujus radix haud plures quam ($2n$) valores habet; duo vero ſinus in eandem inclinationem referuntur, & conſequenter ſolummodo poſſunt eſſe n inclinationes, in quibus ordinatæ curvam (n) dimen-

menſionum haud ſecant in (n) punctis vel poſſibilibus vel impoſſibilibus, & conſequenter haud plures poſſunt eſſe abſciſſæ, vel diametri ſecundi generis quam n.

Ut fiat diameter vel abſciſſa tertii generis, neceſſe eſt ut evaneſcant prædictus terminus (v^n) & termini ($z\,v^{n-1}$ & v^{n-1}); coefficientes vero horum terminorum reſpective ſunt, $bS^{n-1} + 2eS^{n-2}\sqrt{1-S^2} + 3k S^{n-3} \times \overline{1-S^2}$ + &c. & $aS^{n-1} + dS^{n-2}\sqrt{1-S^2} + bS^{n-3} \times \overline{1-S^2}$ + &c. Fiant hæ coefficientes nihilo reſpective æquales, tum reſultant duæ æquationes haud plurium quam ($2 \times \overline{n-1}$) dimenſionum, conſequenter numerus valorum incognitæ quantitatis (S) haud major eſt quam $2 \times \overline{n-1}$; exinde numerus diverſarum inclinationum ad abſciſſam AP haud majores erit quam $n-1$; ſed ſecundæ æquationis radices ſunt limites inter primæ æquationis radices, & conſequenter non plures poſſunt eſſe diverſæ radices harum duarum æquationum inter ſe æquales, quam $\frac{n}{2}$.

Terminorum ($z^2 v^{n-2}$, $z v^{n-2}$, v^{n-2}) coefficientes ſunt reſpective $eS^{n-2} + 3kS^{n-3}\sqrt{1-S^2}$ + &c. $dS^{n-2} + 2bS^{n-3}\sqrt{1-S^2}$ + &c. $cS^{n-2} + gS^{n-3}\sqrt{1-S^2}$ + &c. Fiant hæ coefficientes & prima $S^n + bS^{n-1}\sqrt{1-S^2} + eS^{n-2} \times \overline{1-S^2} + kS^{n-3} \times \overline{1-S^2}^{1\frac{1}{2}}$ + &c. nihilo reſpective æquales, tum ex hâc & primâ æquatione ſequitur haud plures diverſas radices eſſe harum æquationum inter ſe æquales quam $\frac{n}{3}$; & ſic deinceps.

Non poſſunt igitur eſſe plures quam $\frac{n}{m}$ inclinationes parallelarum ordinatarum, quæ ſecant curvam ſolummodo in ($n-m$) punctis.

Non poſſunt eſſe plures quam (n) diverſæ ordinatarum inclinationes curvam in (n) punctis vel poſſibilibus vel impoſſibilibus haud ſecantium.

Hoc in loco impoſſibiles ſectiones eodem modo ac poſſibiles habentur.

Cor.

Cor. Eadem linea haud plures habet quam $\frac{n}{m+1}$ diverſas inclinationes ordinatarum, quæ ſolummodo ſecant curvam in $n-m$, $n-m-1$, $n-m-2$, &c. punctis, quarum ordinatarum reſpective linea prædicta eſt diameter.

THEOR. VI.

Fig. 6. Sit curva (MNS), cujus linea (AP) eſt diameter, & cujus ordinatæ utrinque ſunt inter ſe æquales, i. e. negativi & affirmativi ordinatæ valores inter ſe reſpective ſunt æquales, tum æquatio erit hujus formulæ $y^{2n} + \overline{c+dx+ex^2}\, y^{2n-2} + \overline{f+gx+hx^2+kx^3+lx^4} \times y^{2n-4}$ $+$ &c. $=0$. Ita ſcribatur hæc æquatio, ut termini progrediantur ſecundum dimenſiones incognitarum quantitatum $(x$ & $y)$ in iis contentarum, $\alpha + \beta x + \gamma y^2 + \delta x^2 + \epsilon x y^2 + \zeta x^3 + n y^4 + \theta x^2 y^2 + i x^4$ $+$ &c. $=0$, transformetur hæc æquatio, ut relationem inter abſciſſam $Ap\,(z)$ & ejus ordinatas $pM\,(v)$ rectum angulum cum abſciſsâ (Ap) facientes, exprimat; i. e. ſi modo (q) ſit ſinus anguli PAp, pro PM (y) ſubſtituatur $qz + \sqrt{1-q^2} \times v$, & pro $AP\,(x)$ ſcribatur $\sqrt{1-q^2}\,z$ $-\,qv$; quibus quantitatibus pro ſuis valoribus ſubſtitutis, & reſultantis æquationis terminis, in quibus inveniuntur impares ordinatæ (v) dimenſiones, nihilo æqualibus eſſe ſuppoſitis, reſultant æquationes, e quibus cognoſci poſſunt diametri. e. g. Terminus unius dimenſionis eſt (βx), in eo pro (x) ſcribatur $\sqrt{1-q^2}\,z - qv$, reſultat $\beta\sqrt{1-q^2}\,z - \beta vq$; ſed v eſt impar ordinatæ dimenſio, & conſequenter $\beta q v$ nihilo debet eſſe æqualis, & exinde vel 1mo ſit β data quantitas, & conſequenter $q=0$, tum curva non habet aliam diametrum præter datam; vel 2do, ſit $\beta = 0$, tum nulli ſunt termini, in quibus una ſolummodo continetur dimenſio.

In terminis duarum dimenſionum $(\gamma y^2 + \delta x^2)$ pro $(x$ & $y)$ ſcribantur prædicti valores, tum reſultant

$$\left.\begin{matrix}\gamma \times \overline{1-q^2} \\ \delta \times q^2\end{matrix}\right) v^2 \left.\begin{matrix}+2\gamma q \\ -2\delta q\end{matrix}\right) \sqrt{1-q^2}\, zv \left.\begin{matrix}+\gamma q^2 \\ +\delta \times \overline{1-q^2}\end{matrix}\right) z^2.$$

Ut fiat (Ap) diameter vel axis, terminus $2 \times \overline{\gamma-\delta} \times q\sqrt{1-q^2}\, zv$, in quo impar eſt dimenſio ordinatæ, debet eſſe nihilo æqualis. Si vero $\overline{\gamma-\delta} \times q\sqrt{1-q^2}$ nihilo

nihilo eſt æqualis; tum, ſi $\overline{\gamma^2-\delta}$ ſit data quantitas, vel q vel $\sqrt{\overline{1-q^2}}$ nihilo eſt æqualis, & q vel $= 0$ vel $=\pm 1$, i. e. quoad hoc curva poteſt habere duas diametros ſecum angulum rectum facientes: ſi $\gamma-\delta=0$, tum quoad hoc curva poteſt habere infinitas hujuſmodi diametros; & hic caſus verus eſt in circulis, in quibus infinitæ dantur diametri.

In terminis $(\varepsilon x y^2+\zeta x^3)$ pro $(x$ & $y)$ ſcribantur prædicti valores, tum reſultant

$$\left.\begin{matrix}-\varepsilon q\times\overline{1-q^2}\\-\zeta\times q^3\end{matrix}\right)v^3+\left.\begin{matrix}\varepsilon\times\overline{1-3q^2}\\+3\zeta q^2\end{matrix}\right)\sqrt{\overline{1-q^2}}\,zv^2+\left.\begin{matrix}\varepsilon\times\overline{2q-3q^3}\\-3\zeta q\times\overline{1-q^2}\end{matrix}\right)z^2v+\left.\begin{matrix}\varepsilon\times q^2\,\overline{1-q^2}|^{\frac{1}{2}}\\+\zeta\times\overline{1-q^2}|^{1\frac{1}{2}}\end{matrix}\right)z^3,$$

ſed ut fiat diameter, neceſſe eſt, ut evaneſcant termini $(\overline{\varepsilon q\times\overline{1-q^2}+\zeta q^3}\,v^3,\ \overline{2\varepsilon q-3\varepsilon q^3-3\zeta q\times\overline{1-q^2}}\,z^2v)$, in quibus inveniuntur impares ordinatæ (v) dimenſiones; e priori vero termino ſequitur $\varepsilon=\frac{-\zeta q^2}{1-q^2}$, quâ quantitate pro ejus valore in ſecundâ æquatione ſubſtitutâ, reſultat æquatio $\frac{-\zeta q^2}{1-q^2}\times\overline{2q-3q^3}-3\zeta q\times\overline{1-q^2}=0$, quâ reductâ ſequitur vel $q=0$, vel $q^2=\frac{3}{4}$, & $q=\sqrt{\frac{3}{4}}=$ ſin. 60°; & conſequenter, ſi in datâ æquatione ſint quicunque termini tres habentes dimenſiones abſciſſæ & ordinatæ, tum non poteſt habere curva plures quam tres diametros; & ſi tres ſint diametri, ſeſe interſecant in angulo 60°; & ε vel $=-3\zeta$ vel ζ; poſterior pars hoc modo deduci poteſt; ex æquatione $\varepsilon q\times\overline{1-q^2}+\zeta q^3=0$, conſtat $\frac{\varepsilon}{\varepsilon-\zeta}=q^2$; quo valore in æquatione $\varepsilon\times\overline{2-3q^2}-3\zeta\times\overline{1-q^2}=0$ ſubſtituto, reſultat æquatio $\varepsilon\times\overline{2-\frac{3\varepsilon}{\varepsilon-\zeta}}-3\zeta\times\overline{1-\frac{\varepsilon}{\varepsilon-\zeta}}=0$, quâ reductâ fit $\varepsilon^2+2\zeta\varepsilon-3\zeta^2=0$, cujus radices (ε) ſunt vel $=-3\zeta$ vel $=\zeta$; cum $\varepsilon=\zeta$, tum $q^2=\frac{\varepsilon}{\varepsilon-\zeta}=\frac{\varepsilon}{0}=$ infin.

In terminis $(ny^4+\theta y^2x^2+ix^4)$ quatuor dimenſionum ſubſtituantur pro $(x$ & $y)$ eædem quantitates, quæ prius ſubſtitutæ fuerunt, & quantitatum reſultantium fiant illæ, in quibus impares inveniuntur

ordinatæ

ordinatæ dimenſiones nihilo æquales; & exinde ſequitur ſi $2n = 2i = 0$; ~~tum~~ quoad hos terminos infinitas dari poſſe diametros; ſin illæ quantitates haud ſunt inter ſe æquales, tum non plures quam quatuor poſſunt eſſe diametri, & ſi quatuor ſunt diametri, angulum inter ſe faciunt $45°$ graduum.

Duæ poſſunt eſſe in hoc caſu ſolummodo diametri; ſed hæc reſtrictio oritur e terminis duarum dimenſionum abſciſſæ & ordinatæ.

In genere ſint curvæ n dimenſionum, quæ (m) habent diametros; & anguli, quos hæ diametri inter ſe faciunt, erunt $= \frac{180°}{m}$.

Sint termini $2n$ dimenſionum in datâ æquatione $\overline{x^2 + y^2}^n$, & quoad hos terminos infinitæ dari poſſunt diametri.

Si modo n eſt primus numerus, tum habet unam vel n diametros vel nullam omnis algebraica curva: & curva imparium dimenſionum non poteſt habere parem diametrorum numerum.

Si curva algebraica habeat m æquales partes, tum æquationis ad diametrum & ejus ordinatas termini debent eſſe rationales functiones quantitatum $x^2 + y^2$, & $x^m - m.\frac{m-1}{2}x^{m-2}y^2 + m.\frac{m-1}{2}.\frac{m-2}{3}.\frac{m-3}{4}.x^{m-4}y^4 -$ &c. & $m\,x^{m-1}y - m.\frac{m-1}{2}.\frac{m-2}{3}x^{m-3}y^3 + m.\frac{m-1}{2}.\frac{m-2}{3}.\frac{m-3}{4}.\frac{m-4}{5}x^{m-5}y^5 -$ &c.

Si alterutra poſteriorum expreſſionum non ingrediatur in æquationem, curva habet m diametros.

Si curva duas vel plures habet diametros, tum generale habet centrum, & omnes lineæ per hoc centrum ductæ erunt utrinque inter ſe æquales.

Si ducantur etiam per centrum lineæ æquales angulos cum primo, tertio, quinto, &c. axe facientes, tum hæ diametri æquales angulos cum ſuis ordinatis faciunt.

Primus, tertius, quintus, &c. axes erunt inter ſe æquales.

Eodem

Eodem modo quo inveniuntur axes, deduci poſſunt diametri earum ordinatas ad datum angulum ſecantes, & ſic deinceps.

Fere omnia ea, quæ prius tradita fuere, e geometricis principiis facile demonſtrari poſſunt, quod facile conſtabit e ſubſequentibus.

Fig. 65. Sit data curva $ABDn$, &c. ſint CA & CB duo diametri datæ curvæ; & ſit angulus $BCA =$ ang. DCB; tum CD erit etiam diameter & diametro AC æqualis.

Ducatur enim ordinata Ae ab puncto A ad rectum angulum ad BC, & quoniam ordinatæ utrinque biſecantur, erit $eD = Ae$; unde triangula AeC & DeC habent latera Ae, eC & eD, eC inter ſe reſpective æqualia, & angulos AeC & CeD rectos, ergo reliqua latera CD & CA erunt inter ſe æqualia.

Quod CD erit diameter, ſic probari poſſit.

Ducatur linea fgh ordinata ad diametrum, AC, & erit $fg = gh$; aſſumatur $Cm = Cg$, & ducatur linea lmn ad rectum angulum ad lineam Cm in puncto m & capiatur $lm = mn = fg = gh$; ducatur linea hl; & quoniam lineæ Cg & Cm, hg & lm ſunt inter ſe æquales, & anguli Cgh & Cml recti, erunt latera hC & lC & anguli lCm & hCg, & conſequenter anguli lCk & hCk inter ſe æqualia; ſed duo triangula hCk & kCl habent latera hC, kC & lC, kc reſpective & angulos lCk & hCk inter ſe æqualia; ergo anguli hkC & lkC erunt recti, & reliqua latera hk & kl inter ſe æqualia; & quoniam CB eſt diameter & punctum h ponitur ad curvam, punctum l etiam erit ad curvam poſitum, & ſic probari poſſit punctum n ad curvam eſſe poſitum, & conſequenter lineam CD eſſe diametrum, biſecat enim ſuas ordinatas.

Si omnes lineæ per punctum, a quo incipiat abſciſſa, ductæ, in æquales partes dividantur; tum æquatio relationem inter abſciſſam & ejus correſpondentes ordinatas erit hujuſce formulæ $y^n + bxy^{n-1} + \overline{c+dx^2}\,y^{n-2} + \overline{cx+fx^3}\,y^{n-3} +$ &c. $= o$, in quâ dimenſiones terminorum nunquam minores erunt quam datæ æquationis dimenſiones per imparem numerum.

THEOR.

THEOR. VII.

Fig. 2. Sunt curvæ n dimenſionum, quæ ſecare poſſunt rectam lineam in (n) punctis; ſunt etiam curvæ $(2n)$ parium dimenſionum, quæ nullam interſecant rectam lineam in pluribus quam (2, 4, 6, &c.) punctis; & imparium dimenſionum, quæ nullam interſecant rectam in pluribus quam (3, 5, 7, &c.) punctis; ſed obſervandum eſt, quod par erit maximus numerus punctorum, in quibus recta linea ſecare poteſt curvam parium dimenſionum, & impar imparium dimenſionum, e. g. Fig. 2. ſit curva parabolica, cujus æquatio deſignans relationem inter abſciſſam $AP = x$ & ejus ordinatas $PM = y$, ſit $y^n = px$; tum, ſi n ſit impar numerus, recta linea ſecare poteſt curvam in tribus & haud pluribus punctis. Si enim poſſibile ſit, ducatur linea pM curvam ſecans in pluribus quam tribus punctis; ad punctum curvæ M ducatur ordinata $PM = y$, & ſint ſinus angulorum MPp, MpP & pMP reſpective 1, q & r; & linea pM, v; & Ap, z; tum per Cor. 1. Prob. 1. vel per trigonometriam invenietur $qv = y$ & $z + rv = x$, quibus valoribus pro x & y in datâ æquatione ſubſtitutis, reſultat æquatio quæſita $q^n v^n = prv + pz$, ſed e Medit. Algeb. conſtat æquationem $v^n = Av + B$ haud plures quam tres poſſibiles radices habere poſſe; ergo linea $pM = v$, cujus æquatio ſit $q^n v^n = prv + pz$, haud plures habet quam tres poſſibiles valores, & conſequenter linea pM haud ſecat curvam in pluribus quam tribus punctis.

Si n ſit par numerus, eodem modo probari poteſt, quod haud ſecat curvam in pluribus quam duobus punctis.

Et ſic ratiocinari licet de interſectionibus prædictarum curvarum & conicarum ſectionum, &c.

THEOR. VIII.

Fig. 7. Sit curva McM, &c. cujus æquatio relationem inter abſciſſam (AP) & ejus correſpondentes ordinatas (PM) exprimens eſt $Ay^n + \overline{a + bx}\, y^{n-1} + \overline{c + dx + ex^2} \times y^{n-2} + \&c. = 0$; ſint termini,

 in

in quibus maximæ inveniuntur dimenſiones incognitarum quantitatum (y & x), $Ay^n + bxy^{n-1} + ex^2y^{n-2} +$ &c. ducantur duæ aliæ lineæ (PC, Pc) curvam in (n) punctis & ſeſe interſecantes in puncto abſciſſæ (P), & cum eâ angulos facientes, quorum ſinus ſint q & r reſpective; ſit 1 ſinus anguli (MPA), quem ordinatæ cum diametro faciunt, & C, c ſinus angulorum quos lineæ (PC, Pc), ſeſe interſecantes in puncto (P) cum ordinatis (PM) faciunt, tum contentum ſub (n) lineis inter punctum (P) & curvam in directione Pc interceptis, erit ad contentum ſub (n) lineis inter punctum P & curvam in directione (PC) interceptis in eâdem ratione, quam habet $Aq^n + bq^{n-1}C + eq^{n-2}C^2 +$ &c. ad $Ar^n + br^{n-1}c + er^{n-2}c^2 +$ &c.

Cor. Abſciſſa ſit axis curvæ, cujus omnes diametri ſint parallelæ, tum nihilo evadent æquales b, e, &c. & prædicta ratio $Aq^n + bq^{n-1}C +$ &c. : $Ar^n + br^{n-1}c +$ &c. erit $Aq^n : Ar^n$.

Et ſic ratiocinari licet de rationibus, quas inter ſe habent ſummæ quadratorum, cuborum vel quarumcunque functionum ex abſciſſis & ordinatis quâcunque datâ methodo deductarum.

PROB. IV.

Datis duabus æquationibus eaſdem dimenſiones habentibus, & relationes inter abſciſſas x & z duarum curvarum & earum correſpondentes ordinatas y & v exprimentibus, invenire, annon hæ duæ curvæ ſunt eædem.

Pro x & y in una æquatione ſubſtituantur reſpective $nz + qv + r$ & $mz + ov + p$; & fiant tum reſultantis tum alterius æquationis correſpondentes coefficientes eædem, ſi hoc poſſibile ſit: & per prob. 1. inveniatur, utrum abſciſſæ & ejus ordinatarum transformatio admittat hujuſmodi ſubſtitutionem necne, & conficitur problema.

Et ſimiliter inveniri poteſt, annon duæ curvæ datam inter ſe habeant relationem, & innumeras hujuſmodi propoſitiones adjicere licuiſſet.

PROB.

PROB. V.

Datâ æquatione relationem inter curvæ abſciſſam & ejus ordinatas exprimente, invenire ejus crura in infinitum abeuntia.

Per infinitas ſeries inveniantur ſinguli ordinatæ (y) valores terminis vero infinitis ſecundum abſciſſæ x dimenſiones progredientibus, ita ut in ſingulis ſucceſſivis terminis abſciſſæ x dimenſiones fiant continuo minores, i. e. ſit $y = Ax^m + Bx^{m-r} +$ &c. $+ P + \frac{Q}{x^s} +$ &c. Crus, quod deſignat hæc æquatio erit hyperbolicum, ſi modo ſit $y = Ax + P + \frac{Q}{x^s} +$ &c. cujus æquatio ad aſymptoton erit $v = Ax + P$; ſin aliter erit parabolicum, cujus parabolici cruris ordo pendet e terminis $Ax^m + Bx^{m-r} +$: e. g. ſit $m = 2$ vel $\frac{1}{2}$, &c. crus erit parabolicum conici generis, &c.

Cor. 1. Sit æquatio relationem inter abſciſſam (x) & ejus correſpondentes ordinatas y exprimens $y^n + \overline{a+bx}\,y^{n-1} + \overline{c+dx+ex^2}\,y^{n-2}$ + &c. $= 0$. Sint termini, in quibus maximæ inveniuntur ordinatæ & abſciſſæ (x & y) dimenſiones $y^n + bxy^{n-1} + ex^2 y^{-2} +$ &c. fiant hi termini nihilo æquales, & reſultat $y^n + bxy^{n-1} + ex^2 y^{n-2} +$ &c. $= 0$. Sint omnes radices hujus æquationis impoſſibiles, & curva nullum habet crus in infinitum progrediens. Sint $2m$ radices impoſſibiles, & ($n-2m$) poſſibiles radices ordinatæ (y) αx, βx, γx, δx, &c. reſpective: i. e. $y = \alpha x +$ &c. $y = \beta x +$ &c. $y = \gamma x +$ &c. $y = \delta x +$ &c. & conſequenter curva habet ($n-2m$) aſymptotos, ($2n-4m$) hyperbolica crura in infinitum progredientia, habentes.

Sint poſſibiles radices inter ſe æquales, & eodem modo e ſubſequentibus terminis datæ æquationis inveniri poteſt formula ſeriei, a quâ deſignatur ordinata (y), e. g. ſint duæ radices prædictæ αx & βx æquales, & hæ duæ æquales radices vel deſignant duas aſymptotos eſſe eaſdem vel parallelas, vel quatuor hyperbolica crura in crura conicæ parabolæ transformari, vel penitus evaneſcere, i. e. haud in infinitum progredi.

Subſtituatur enim pro y novus valor aſſumptus $pz + q$ in datâ æquatione ($y^n + \overline{a+bx}\, y^{n-1} + \&c. = o$), & ſi proximus valor aſſumptæ quantitatis (q) inveniatur $fx^{\frac{1}{2}}$, tum crus deſignat parabolicum conici generis; ſi vero ſit $\sqrt{-g^2}$, tum deſignat nullum crus exinde in infinitum progredi; ſin aliter, hyperbolica crura denotat, & ſic deinceps.

Cor. 2. Sit $n = m + 2o + 3r + 4s + 5t + \&c.$ & facile deduci poteſt curva (n) dimenſionum quaſcunque (m) lineas aſymptotos, quæcunque o parabolica conici generis crura, (r) ſecundi generis parabolica, & ſic deinceps habens.

Fig. 2. Sit enim AP (x) abſciſſa, & PM (y) ejus correſpondens ordinata, & ſint æquationes ad aſymptotos $y + px + q = o$, $y + p'x + q' = o$, $y + p''x + q'' = o$, &c. & æquatio ad parabolam conici generis $\overline{y + \alpha x}^2 + \beta y + \gamma x + \delta = o$, & ſic deinceps: e multiplicatione horum terminorum in ſeſe conſtant dimenſiones, ad quas aſſurgit data æquatio, & termini ipſi, in quibus inveniuntur n & $n - 1$ dimenſiones, &c.

PROB. VI.

Datâ æquatione relationem inter abſciſſam & ejus correſpondentes ordinatas exprimente $Ay^n + \overline{a+bx}\, y^{n-1} + \overline{c+dx+ex^2}\, y^{n-2} + \&c. = o$. invenire, qui ſit valor abſciſſæ, quando duo ordinatæ (y) valores fiunt inter ſe æquales.

Si duo ordinatæ valores ſunt inter ſe æquales, tum $nAy^{n-1} + \overline{n-1}\; \overline{a+bx}\, y^{n-2} + \overline{n-2} \times \overline{c+dx+ex^2} \times y^{n-3} + \&c. = o$, transformentur hæ duæ æquationes in unam, ita ut exterminetur incognita quantitas (y), & reſultat æquatiò, cujus radix eſt abſciſſa, quando duo ordinatæ valores fiunt inter ſe æquales.

Subſtituantur hæ radices pro ſuo valore (x) in duabus datis æquationibus, & æquationum reſultantium communes diviſores erunt valores ordinatarum, quando duo ſunt inter ſe æquales.

Cor. 1.

Cor. 1. Curvæ crura, cujus æquatio deſignans relationem inter ejus abſciſſam & ordinatas eſt $nAv^{n-1} + \overline{n-1} \times \overline{a+bx}\,v^{n-2} + \overline{n-2}\,\overline{c+dx+ex^2}\,v^{n-3} +$ &c. $= o$, ſemper interponantur inter crura curvæ, cujus æquatio relationem inter abſciſſam & ejus ordinatas exprimens, eſt $Ay^n + \overline{a+bx}\,y^{n-1} + \overline{c+dx+ex^2}\,y^{n-2} +$ &c. $= o$.

Cor. 2. Hinc inveniri poteſt, quando duo, quatuor, &c. ordinatæ valores fiunt impoſſibiles.

Inveniatur enim, quando duo valores fiunt æquales, & quando duo bis fiunt æquales, & ſic deinceps; exinde inveniri poteſt, quando duo, quatuor, &c. ordinatæ valores fiunt impoſſibiles.

Cor. 3. Hinc etiam inveniri poſſunt puncta duplicia, triplicia, &c. & puncta conjugata, & ovales.

Inveniatur, quando duo, tres, quatuor, &c. ordinatæ (y) valores fiunt inter ſe æquales.

Si duo, quatuor, ſex, &c. ordinatæ valores fiunt inter ſe æquales, & utrinque ſunt correſpondentes ordinatæ impoſſibiles, tum erit purum punctum conjugatum, ſimplex, duplex, &c.

Sint præcedentes ordinatæ utrinque poſſibiles, & curva habet punctum duplex, quadruplex, &c.

Si ordinatæ partim ſunt poſſibiles, & partim impoſſibiles utrinque, tum ſingula ordinata utrinque numerum æqualium radicum unitate auget; ſi vero ordinata tangit curvam, tum auget numerum æqualium ordinatarum numero binario; & ſic de punctis contrariæ flexuræ, cuſpidibus, &c.

Si plures ſunt ordinatæ æquales, quam requirit figura, tum datur punctum ad curvam. Fig. 8.

Si duæ vel plures ordinatæ bis fiunt æquales, quarum abſciſſæ reſpective ſunt $P'A$ & PA, & inter eas ſint ordinatæ poſſibiles: ordinatæ vero PM & $P'M'$ eundem habeant numerum crurum ab utrâque parte punctorum M & M' jacentium, tum jacet inter puncta M & M' ovalis.

Cor. 4. Sit ovalium & punctorum numerus, in quibus duo, tria, quatuor, &c. puncta coeant, reſpective (b & m, r, s, t, &c.) & N vel $= \sqrt{2b+2m+2r+2s+\text{\&c.}+\frac{9}{4}} - \frac{3}{2}$, ſi ſit integer numerus, vel integer numerus ei proxime ſuperior, ſi modo ſit ſurda quantitas; tum dimenſiones (M) curvæ, in quâ includuntur hæc puncta, ovales, &c. non poſſunt

poſſunt eſſe minores quam $\frac{N \times \frac{N+3}{2} + b + m + 2r + 3s + 4t + \&c.}{N}$: Curva enim N dimenſionum, i. e. $N-1$ ordinis duci poteſt per quæcunque $N \times \frac{N+3}{2}$ puncta per Prob. 11. Cor. 8. & conſequenter duci poteſt per $N \times \frac{N+3}{2}$ puncta curvæ $M-1$ ordinis, ducatur igitur ea per prædictas b ovales, & m, r, s, &c. puncta curvæ $M-1$ ordinis; hinc duæ curvæ ſeſe interſecant in $N \times \frac{N+3}{2} + b + m + 2r + 3s + 4t + \&c. = H$ punctis; ſed duæ curvæ N & M dimenſionum haud ſeſe interſecare poſſunt in pluribus quam $N \times M$ punctis, nam duæ incognitæ quantitates duarum æquationum N & M dimenſionum haud plures habere poſſunt valores quam $N \times M$ reſpective, ergo $N \times M$ haud minor eſſe poteſt quam H, & conſequenter M haud minor erit quam $\frac{H}{N}$. *Q. E. D.*

PROB. VII.

Datâ æquatione $Ay^n + \overline{a+bx}\,y^{n-1} + \overline{c+dx+ex^2}\,y^{n-2} + \&c. = 0$, relationem inter abſciſſam (x), & ejus ordinatas (y) exprimente, invenire æquationem relationem inter quaſcunque duas vel plures alias lineas quocunque modo ex prædictis derivatas exprimentem.

Inveniantur æquationes relationem inter abſciſſam, ejus ordinatas & lineas, inter quas requiritur relatio, exprimentes; transformentur hæ æquationes, ita ut exterminentur omnes incognitæ quantitates præter eas lineas vel quantitates, inter quas requiritur relatio, & conſit problema.

Fig. 9. Ex. datâ prædictâ æquatione invenire æquationem relationem exprimentem inter lineas $(EM$ & $FM)$ a quibuſcunque duobus punctis $(E$ & $F)$ ad quodlibet curvæ punctum M, ductas.

Sit AP (x) abſciſſa, & PM (y) ordinata; per puncta E & F ducantur lineæ EK & FL parallelæ ordinatis (PM) & occurrentes abſciſſæ in punctis K & L, ſcribantur pro $KA, EK; LA, FL; EM, FM$ reſpective E, F, G, H, v, z; & pro ſinu anguli (MPA) & ejus coſinu s & c reſpective

reſpective, tum $EM^2 = s^2\times\overline{EK+PM}^2 + \overline{c\times\overline{PM+EK}+PK}^2$ & $FM^2 = s^2\times\overline{FL+PM}^2 + \overline{c\times\overline{PM+FL}+LP}^2$, pro quibus lineis ſubſtituantur earum valores, & reſultant duæ æquationes $v^2 = y^2 + \overline{2F-2cE+2cx}\,y + F^2 - 2cFE + E^2 + \overline{2cF-2E}\times x + x^2$ & $z^2 = y^2 + \overline{2H-2cG+2cx}\,y + H^2 - 2cHG + G^2 + \overline{2cH-2G}\,x + x^2$, ex his duabus æquationibus inveniantur valores incognitarum quantitatum (x & y) terminis vero incognitarum quantitatum (z & v), quibus inventis & pro ſuis valoribus (x & y) in datâ æquatione ſubſtitutis, reſultat æquatio relationem inter ($v = EM$ & $z = FM$) exprimens.

2. Ita transformetur per Cor. 1. Prob. 1. ordinatarum inclinatio, ut angulus MPA, quem ordinatæ PM cum ſuâ abſciſsâ AP faciunt, ſit rectus, & ejus coſinus (c) nihilo erit æqualis; tranſeat etiam abſciſſa per puncta E & F; i. e. F & H nihilo ſint æquales, tum pro x ſubſtituatur $\frac{z^2-v^2-G^2+E^2}{2\times\overline{E-G}}$, & pro y^2 ſcribatur $v\ \overline{\frac{z^2-v^2-G^2+E^2}{2\times\overline{E-G}}}^2 - E$, & reſultat æquatio quæſita.

Cor. 1. Hinc conſtat dimenſiones æquationis relationem inter lineas EM & FM exprimentis non plures eſſe quam dimenſiones æquationis relationem inter abſciſſam & ejus correſpondentes ordinatas exprimentis quater ſumptas. Et vice verſâ non poſſunt dimenſiones æquationis relationem inter abſciſſam & ejus correſpondentes ordinatas exprimentis eſſe plures quam dimenſiones æquationis relationem inter lineas EM & FM exprimentis, quater ſumptæ.

Cor. 2. Si nullæ impares dimenſiones linearum z & v contineantur in datâ æquatione curvæ relationem inter lineas z & v exprimente, tum dimenſiones æquationis relationem inter abſciſſam (x) & ejus ordinatas (y) exprimentis haud majores eſſe poſſunt quam dimenſiones datæ æquationis.

Eodem modo, ſi lineæ, (z, v, w, n, &c.) a quocunque curvæ puncto ducantur ad plura data puncta, & nullæ impares dimenſiones linearum (z, v, w, n, &c.) in datâ æquatione contineantur, tum æquatio relationem inter abſciſſam (x) & ejus correſpondentes ordinatas (y) exprimens non majores habere poteſt dimenſiones, quam habet data æquatio, e. g. ſi ſumma quadratorum e ſingulis prædictis lineis ſit

data

data quantitas, tum æquatio relationem inter abſciſſam & ejus correſpondentes ordinatas exprimens non plures habet quam duas dimenſiones, i. e. curva erit conica ſectio, & facile conſtat eam eſſe circulum.

Et ſic de pluribus hujuſce generis proprietatibus.

Cor. 3. Sit æquatio relationem inter lineas $(z$ & $v)$ prædictas exprimens $v^n - \overline{a+bz}\,v^{n-1} + \overline{c+dz+ez^2}\,v^{n-2} +$ &c. $= 0$, & facile e vulgari algebra conſtant propoſitiones iis conſimiles, quæ ex æquatione algebraicâ relationem inter abſciſſam & ejus ordinatas exprimente vulgariter innoteſcunt, e. g. ſi z ſit data quantitas A, tum ſumma e ſingulis valoribus lineæ (v) ſemper erit $a+b$, ſi vero hæc ſumma fiat bis nihilo æqualis, tum ſemper erit nihilo æqualis; &c. & ſic inveniri poſſunt curvæ aſymptoti, ovales, &c.

Cor. 4. Data æquatione inter lineas $(z$ & $v)$ relationem exprimente, facile ad modum exempli prioris inveniri poteſt æquatio relationem inter abſciſſam x & ejus correſpondentes ordinatas (y) exprimens.

Et ſic de lineis ad plura puncta ductis, & proprietatibus exinde deducendis.

Hæc etiam ad fluxionales curvas applicari poſſunt.

PROB. VIII.

Fig. 11. Datâ æquatione relationem inter abſciſſam (AP) & ejus correſpondentes ordinatas (PM), vel inter quaſcunque alias lineas, quæ curvam definiunt, exprimente; invenire æquationem relationem inter abſciſſam $(A\pi)$ & ejus ordinatas (πm) deſignantem, quum loci punctorum (m) per quemcunque datum modum e datâ æquatione deducantur.

Datâ methodo inveniendi punctum m e datâ æquatione, datur etiam methodus inveniendi lineas $(A\pi$ & $\pi m)$ abſciſſam & ordinatam quæſitæ curvæ; æquationes reſultantes ita reducantur in unam, ut exterminentur omnes incognitæ quantitates præter duas $(A\pi$ & $\pi m)$ abſciſſam & ordinatam quæſitæ curvæ, & confit problema, e. g. datis algebraicis æquationibus relationem inter abſciſſas & earum ordinatas datæ & quæſitæ curvæ exprimentibus, e reductione harum æquationum in unam, ita ut exterminentur omnes incognitæ quantitates præter abſciſſam & ejus correſpondentes quæſitæ curvæ ordinatas; id, quod requiritur, fit.

Ex.

Ex. 1. Sit data æquatio relationem inter abſciſſam (x) & ejus correſpondentes ordinatas (y) exprimens; & ſint z & v reſpective curvæ quæſitæ abſciſſa, & ejus correſpondens ordinata; ſint etiam $x = \frac{pz+qv+r}{Az+Bv+C}$ & $y = \frac{Pz+Qv+R}{Az+Bv+C}$; ſubſtituantur hæ quantitates pro ſuis valoribus (x & y) in datâ æquatione, reſultat æquatio relationem inter novam abſciſſam (z) & ejus ordinatas (v) curvæ exprimens, cujus dimenſiones æquales erunt datæ æquationis dimenſionibus.

Et ſimiliter pro x & y in datâ æquatione ſcribantur reſpective $\frac{pz^2+qzv+rv^2+sv+tz+l}{Az^2+Bzv+Cv^2+Dz+Ev+F}$ & $\frac{Pz^2+Qvz+Rv^2+Sv+Tz+l}{Az^2+Bzv+Cv^2+Dz+Ev+F}$, & reſultat æquatio relationem inter abſciſſam z & ejus ordinatas v curvæ, cujus dimenſiones haud majores erunt quam bis dimenſiones datæ æquationis. Et ſic deinceps.

Cor. Hinc e datâ curvâ facile conſtrui poſſunt plurimæ aliæ.

Cor. 1. Datâ æquatione curvæ, invenire æquationem omnium curvarum datæ ſimilium.

Pro abſciſsâ & ordinatâ (x & y) in datâ æquatione ſubſtituantur reſpective nz & nv, & conſit corollarium.

Cor. 2. Invenire æquationem omnium curvarum, quarum abſciſſæ & ordinatæ correſpondentes erunt in datâ ratione ad abſciſſas & ordinatas curvæ, cujus datur æquatio. Subſtituantur pro abſciſsâ (x) & ordinatâ (y) in æquatione ad datam curvam reſpective nz & mv, & conſit corollarium.

Ex. 2. Datâ æquatione relationem inter abſciſſam (x) & ejus correſpondentes ordinatas exprimente $y^n + Ay^{n-1} + By^{n-2} + Cy^{n-3} +$ &c. $= 0$, in quâ coefficientes (A, B, C, &c.) ſunt reſpective abſciſſæ (x) quæcunque rationales functiones, invenire æquationem relationem definientem inter abſciſſam & ejus correſpondentes ordinatas curvæ, quæ ita ſecat datæ curvæ ordinatas, ut ſummæ cuborum ſingularum ordinatarum jacentium utrinque, ſint inter ſe æquales.

Pro y ſubſtituatur in datâ æquatione $v + Q$, & reſultat æquatio

$$v^n + \overline{nQ + A}\, v^{n-1} + \overline{n.\frac{n-1}{2}Q^2 + \overline{n-1}\,AQ + B}\, v^{n-2} + \overline{n.\frac{n-1}{2}.\frac{n-2}{3}}$$

$\mathcal{Q}^3 + \overline{n-1}.\frac{n-2}{2} A\mathcal{Q}^2 + \overline{n-2} B\mathcal{Q} + C v^{n-3} + \&c. = o$. Summa cuborum e ſingulis incognitæ quantitatis (v) valoribus nihilo fiat æqualis, & per Prob. 1. noſtr. Medit. Algebr. reſultat æquatio quæſita $\overline{n\mathcal{Q}+A}^3 - 3 \times \overline{n\mathcal{Q}+A} \times n.\frac{n-1}{2}\mathcal{Q}^2 + \overline{n-1}\mathcal{Q}A + B + 3 \times n.\frac{n-1}{2}.\frac{n-2}{3}\mathcal{Q}^3 + 3\overline{n-1}.\frac{n-2}{2} A\mathcal{Q}^2 + 3\overline{n-2} B\mathcal{Q} + 3C = o$.

Fig. 11. 2. Ex. 3. Sit æquatio relationem inter abſciſſam $(AP = x)$ & ejus correſpondentes ordinatas $(PM = y)$ exprimens $Ay^n + \overline{a+bx}\, y^{n-1} + \overline{c+dx+ex^2}\, y^{n-2} \ldots\ldots - \overline{px^{n-1} + qx^{n-2} + rx^{n-3} + \&c.} \times y + Px^n + \mathcal{Q}x^{n-1} + Rx^{n-2} + Sx^{n-3} + \&c. = o$. Circa datum punctum (P) gyretur recta (PM), quæ huic curvæ occurrat in (n) punctis $(M, M', M''$, &c.), & ſi in eâdem rectâ ſumatur Pn, ita ut $\frac{1}{Pn} = \frac{1}{PM} \pm \frac{1}{PM'} \pm \frac{1}{PM''} + \&c.$ tum erit locus puncti (n) recta.

Per Cor. 1. Prob. 1. Ita transformetur data æquatio ut exprimat relationem inter abſciſſam AP & ordinatam cum eâ angulum APL vel APN, &c. facientem, cujus ſinus eſt (σ), ſit etiam angulus (APM) rectus; tum in datâ æquatione pro y ſcribatur σv $(PL, PN$, &c); & pro x, z $(AP) + \sqrt{1-\sigma^2}\, v$; & termini reſultantis æquationis, in quibus invenitur nulla & una dimenſio ordinatæ (v), erunt reſpective $Pz^n + \mathcal{Q}z^{n-1} + Rz^{n-2} + \&c.$ & $\overline{p\sigma + nP\sqrt{1-\sigma^2}} \times z^{n-1} + \overline{q\sigma + \overline{n-1}\,\mathcal{Q}\sqrt{1-\sigma^2}} \times z^{n-2} + \overline{r\sigma + \overline{n-2}\,R\sqrt{1-\sigma^2}} \times z^{n-3} + \&c.$ ergo $\frac{1}{Pn} = \frac{1}{PM} \pm \frac{1}{PM'} \pm \frac{1}{PM''} + \&c. = \frac{\overline{p\sigma + nP\sqrt{1-\sigma^2}}\,z^{n-1} + \overline{q\sigma + \overline{n-1}\,\mathcal{Q}\sqrt{1-\sigma^2}}\,z^{n-2} + \overline{r\sigma + \overline{n-2}\,R\sqrt{1-\sigma^2}} \times z^{n-3} + \&c}{Pz^n + \mathcal{Q}z^{n-1} + Rz^{n-2} + Sz^{n-3} + \&c}$: ſint $W\,(Pp)$ & $V\,(pn)$ abſciſſa & correſpondens ordinata lineæ, cujus requiritur æquatio, ſit angulus Ppn rectus, tum $\frac{V}{\sigma} = Pn$ & $\frac{W}{\sqrt{1-\sigma^2}} = Pn$, quibus reductis reſultat $Pn = \sqrt{W^2+V^2}$; & $\sigma = \frac{V}{\sqrt{W^2+V^2}}$;

&

& $\sqrt{1-\sigma^2} = \frac{W}{\sqrt{W^2+V^2}}$, quibus quantitatibus pro ſuis valoribus in æquatione relationem inter Pn, z & σ exprimente ſubſtitutis, reſultat æquatio relationem inter z, V & W exprimens

$$1 = \frac{\overline{pV + nPW}\, z^{n-1} + \overline{qV + \overline{n-1}\, QW}\, z^{n-2} + \overline{rV + \overline{n-2}\, RW}\, z^{n-3} + \&c}{Pz^n + Qz^{n-1} + Rz^{n-2} + Sz^{n-2} + \&c};$$

ſed punctum (P) per hypotheſin datur, & conſequenter $AP = z$, ergo hæc æquatio relationem inter W & V abſciſſam & ordinatam lineæ quæſitæ deſignat, & quoniam in nullis terminis continentur plures quam una dimenſio abſciſſæ vel ordinatæ, linea quæſita erit recta.

Cor. Sit $\frac{1}{Pn^2} = \frac{1}{PM^2} + \frac{1}{PM'^2} + \frac{1}{PM''^2} + \&c.$ & locus punctorum (n) erit conica ſectio, vel duæ rectæ lineæ.

Sit $\frac{1}{Pn}$ vel $\frac{1}{Pn^3} = \frac{1}{PM} + \frac{1}{PM'} + \frac{1}{PM''} + \&c.$ & locus punctorum (n) vel erit curva ſecundi generis, vel conica ſectio & recta linea, vel tres rectæ lineæ; in genere ſit $\frac{a}{Pn^{2r+1}} + \frac{b}{Pn^{2s+1}} + \frac{c}{Pn^{2t+1}} + \&c. = \frac{1}{PM^{2m+1}} + \frac{1}{PM'^{2m+1}} + \frac{1}{PM''^{2m+1}} + \&c.$ ubi r, s, t, &c. integros & vel æquales vel minores numeros quam m; & a, b, c, &c. datas quantitates reſpective denotant; & locus punctorum (n) curva erit haud majorum quam $2n+1$ dimenſionum. Sit $\frac{a}{Pn^{2r}} + \frac{b}{Pn^{2s}} + \frac{c}{Pn^{2t}} + \&c. = \frac{1}{PM^{2m}} + \frac{1}{PM'^{2m}} + \frac{1}{PM''^{2m}} + \&c.$ & locus punctorum n haud erit curva majorum quam $2m$ dimenſionum.

Et ſimiliter de infinitis aliis hujuſmodi propoſitionibus.

E ſubſtitutione præcedenti conſtant hæ propoſitiones.

Fig. 12. Ex. 4. circa datum punctum (A) abſciſſæ verticem revolvatur recta linea AM, quæ occurrat algebraicæ lineæ cujuſcunque ordinis in tot punctis M, &c. quot ſunt ejus dimenſiones, & ſi in eâdem rectâ ſumatur AN, ita ut $AN = AM + AM' + AM'' + \&c.$ tum locus

 punc-

punctorum N poteſt eſſe curva (n) dimenſionum, vel duæ vel plures curvæ, quarum dimenſionum ſumma eſt (n).

Sit æquatio relationem inter abſciſſam $(AP=x)$ & ejus correſpondentes ordinatas $(PM=y)$ exprimens $Ay^n+\overline{a+bx}\,y^{n-1}+\overline{c+dx+ex^2}\,y^{n-2}+\overline{f+gx+hx^2+kx^3}\,y^{n-3}+$ &c. $=0$, ita transformetur hæc æquatio, ut relationem inter abſciſſam (Ap) & ejus ordinatas (pM) exprimat; ſit angulus APM rectus, & pro ſinubus angulorum (ApP, vel AMP, PAp vel PAM) ſcribantur s & $C=\sqrt{1-s^2}$ reſpective, & pro lineis (Ap & pM) z & v, tum reſultant quantitatum x & y valores sz & $v+\sqrt{1-s^2}\,z$, quibus quantitatibus pro ſuis valoribus ſubſtitutis, coefficientes terminorum (z^n & z^{n-1}) reſultantis æquationis erunt reſpective $AC^n+bC^{n-1}s+eC^{n-2}s^2+kC^{n-3}s^3+$ &c. & $aC^{n-1}+dC^{n-2}s+hC^{n-3}s^2+$ &c. & conſequenter ſumma radicum (z) erit $-\frac{aC^{n-1}+dC^{n-2}s+hC^{n-3}s^2+\&c.}{AC^n+bC^{n-1}s+eC^{n-2}s^2+kC^{n-3}s^3+\&c.}=AM+AM'+AM''$ $+$ &c. $=An$; ſint (V & w) reſpective Am & mn, abſciſſa & ejus ordinata curvæ quæſitæ, tum eodem modo, quo in priori exemplo, colligi poteſt $An=\sqrt{w^2+V^2}$, & $s=\frac{V}{\sqrt{w^2+V^2}}$, & $C=\frac{w}{\sqrt{w^2+V^2}}$; quibus quantitatibus pro ſuis valoribus in priori æquatione ſubſtitutis, reſultat æquatio relationem inter abſciſſam Am quæſitæ curvæ & ejus ordinatas mn exprimens $Aw^n+\overline{bV-a}\,w^{n-1}+\overline{eV^2-dV}\,w^{n-2}+\overline{kV^3-nV^2}\,w^{n-3}+$ &c. $=0$, cujus dimenſiones non plures quam n eſſe poſſunt.

Cor. 1. Punctum, circa quod gyretur recta AM, continet $(n-1)$ puncta, e. g. ſit $n=3$, & punctum A ad curvam NKH, &c. erit punctum duplex; ſit $n=4$ & erit punctum triplex; & ſic deinceps; quod conſtabit e ſubſequentibus.

Cor. 2. In æquatione ad curvam NHK, &c. continentur ſolummodo termini n & $n-1$ dimenſionum, unde per ſimplicem æquationem inveniri poſſint curvæ NHK, &c. diverſa puncta.

Cor. 3. Ducatur quicunque parallelarum linearum numerus (mn, $m'n'$, &c.) tranſeuntium per (n) hujuſmodi puncta, i. e. occurrentium

rentium curvæ quæſitæ in n punctis (n, K, H, &c. n' K' H' &c.) & ſi in eâdem rectâ ſemper ſumatur (mo, $m'o'$,&c.) ita ut $mo = mn \pm mK \mp mH \mp$ &c. & $m'o' = m'n' \pm m'K' \mp m'H' +$ &c. erit locus punctorum (o, o', &c.) recta linea.

Et ſic de infinitis hujuſmodi propoſitionibus.

Cor. 3. Sit $AN^r = AM^r \pm AM'^r \pm AM''^r$ &c. curva $HnKn'$ &c. haud majores habet dimenſiones quam $r \times n$, & habet punctum $(r \times \overline{n-1})$ dimenſionum, e. g. ſit $r = 2$ & $n = 3$ & erit $r \times \overline{n-1} = 2 \times 2 = 4$, & conſequenter habet punctum quadruplex data curva.

PROB. IX.

Fig. 14. Ex datâ æquatione $Ay^n + \overline{a + bx}\, y^{n-1} +$ &c. $= o$, relationem inter abſciſſam AP & ejus correſpondentes ordinatas PM exprimente; invenire curvam datæ curvæ centrorum locum.

Sit diameter, cujus abſciſſa (X) incipit a centro; tum æquationis relationem inter abſciſſam X & ejus correſpondentes ordinatas Y exprimentis, nihilo æquales erunt tres termini $(XY^{n-1}, X^{n-1}, Y^{n-1})$.

Ita transformetur data æquatio, ut exprimat relationem inter abſciſſam (Ap, z) & ejus ordinatas (pM, v).

Sit angulus APM rectus, & pro ſinubus angulorum (PAp & ApM) ſcribantur reſpective s & c, erunt ſinus angulorum (prM & rMp) reſpective $\sqrt{1-s^2}$ & $sc + \sqrt{1-s^2} \times \sqrt{1-c^2} = C$, & conſequenter erit abſciſſa $AP = \sqrt{1-s^2}\, z - Cv$, & ordinata $PM = sz + \frac{c - sC}{\sqrt{1-s^2}} v$, quibus quantitatibus pro ſuis valoribus in datâ æquatione ſubſtitutis, in æquatione reſultanti ſcribatur pro abſciſſa (z) ejus valor $X + k$ (AN), & pro ordinata (v) $Y + l$ (Nm), reſultant $AP = \sqrt{1-s^2}\, X - CY + \sqrt{1-s^2}\, k - Cl$ & $PM = sX + \frac{c - Cs}{\sqrt{1-s^2}} Y + sk + \frac{c - Cs}{\sqrt{1-s^2}} l$.

Subſtituantur hæ quantitates pro ſuis valoribus in datâ æquatione, & reſultantis æquationis terminorum (XY^{n-1}, X^{n-1}, Y^{n-1}) inveniantur coefficientes; fiant hæ tres inventæ coefficientes nihilo reſpective æquales,

æquales, & pro coefficiente C ſubſtituatur ejus valor ($sc + \sqrt{1-s^2} \times \sqrt{1-c^2}$), & tres æquationes reſultantes ita reducantur in unam, ut exterminentur duæ incognitæ quantitates (c, C), reſultat æquatio relationem inter AN & NM exprimens; pro AN & NM ſcribantur reſpective $\frac{c - sC}{\sqrt{1-s^2}} u + \frac{C}{c} w$ & $\frac{\sqrt{1-s^2}\, w - s u}{c}$; reſultat æquatio relationem inter abſciſſam As (u) & ejus ordinatas sm (w) lineæ PM parallelas centralis curvæ quæſitæ exprimens.

Æquatio relationem inter abſciſſam & ejus correſpondentes curvæ ordinatas exprimens, cujus locus eſt puncta interſectionum duarum diametrorum ſibi ipſis quam proximarum, e prædictis & incrementorum principiis facile erui poteſt.

PROB. X.

Invenire algebraicarum curvarum proprietates, quæ pendent e dimenſionibus, quas in datâ quantitate habet quæcunque incognita quantitas (x), vel ex æquationis dimenſionibus, cujus radices ſunt quæcunque incognita quantitas (x).

Ex datis incognitæ quantitatis (x) dimenſionibus in datâ quantitate cognoſcitur, e quot & qualibus factoribus in ſeſe ductis componitur data quantitas, &c.

Ex. ſit æquatio $y^n + \overline{a + bx}\, y^{n-1} + \overline{c + dx + ex^2}\, y^{n-2} + \&c. = 0$, relationem inter abſciſſam (x) & ejus correſpondentes ordinatas exprimens, & ſubtangens v erit

$$\frac{ny^n + \overline{n-1} \times \overline{a+bx}\, y^{n-1} + \overline{n-2} \times \overline{c + dx + ex^2}\, y^{n-2} + \&c}{by^{n-1} + \overline{d + 2ex}\, y^{n-2} + \&c}.$$

Sint α, β, γ, δ, &c. ordinatæ (y) diverſi valores, quibus in poſteriori quantitate ſubſtitutis, reſultant ſubtangentes reſpective

$$\frac{n\alpha^n + \overline{n-1}.\,\overline{a+bx}\,\alpha^{n-1} + \overline{n-2}.\,\overline{c+dx+ex^2}.\,\alpha^{n-2} + \&c}{b\alpha^{n-1} + \overline{d + 2ex}\,\alpha^{n-2} + \&c}.$$

$$\frac{n\beta^n + \overline{n-1}.\,\overline{a+bx}\,\beta^{n-1} + \overline{n-2}.\,\overline{c+dx+ex^2}.\,\beta^{n-2} + \&c}{b\beta^{n-1} + \overline{d + 2ex}\,\beta^{n-2} + \&c}.$$ Ducantur

omnes

omnes hi (n) diverſi ordinatæ valores in ſeſe, & aſcendat numerator ad $(n \times n)$ dimenſiones abſciſſæ (x), denominator vero aſſurgat ad $(n \times \overline{n-1})$ dimenſiones; ſupponatur & numerator & denominator nihilo reſpective æqualis, & ſint radices numeratoris & denominatoris reſpective π, ϱ, σ, τ, &c. & ζ, n, θ, i, &c. ſint x & z duo diverſi abſciſſæ valores, & A & B contenta ſub (n) ſubtangentibus ad abſciſſas $(x$ & $z)$ reſpective, tum erit $A : B :: \frac{\overline{x-\pi} \times \overline{x-\varrho} \times \overline{x-\sigma} \times \overline{x-\tau} \times \&c}{\overline{x-\zeta} \times \overline{x-n} \times \overline{x-\theta} \times \overline{x-i} \times \&c} : \frac{\overline{z-\pi} \times \overline{z-\varrho} \times \overline{z-\sigma} \times \overline{z-\tau} \times \&c}{\overline{z-\zeta} \times \overline{z-n} \times \overline{z-\theta} \times \overline{z-i} \times \&c}$.

Cor. Facile ex analyſeos principiis erui poteſt, quot caſus requiruntur ad probandam propoſitionis veritatem.

Et ſic de curvaturæ radiis, & innumeris aliis quantitatibus conſimiles propoſitiones affirmari poſſunt.

THEOR. IX.

Sint duæ curvæ, quarum quicunque termini ſint iidem in æquationibus ad utramque curvam, tum omnis proprietas unius curvæ, quæ ex iis terminis pendet, i. e. deduci poteſt, erit etiam alterius curvæ proprietas.

E. g. Sint termini n & $n-1$ dimenſionum iidem in æquationibus relationes inter abſciſſas & earum ordinatas ad utramque curvam exprimentibus, & transformentur abſciſſæ & ordinatæ utriuſque curvæ, ita ut æquationum reſultantium iidem inveniantur termini n & $n-1$ dimenſionum, ergo omnis proprietas unius curvæ, quæ ſolummodo pendet e terminis n & $n-1$ dimenſionum, erit etiam alterius proprietas.

Cor. 1. Sint duæ curvæ n dimenſionum, i. e. $n-1$ ordinis, quarum termini n dimenſionum in utrâque æquatione relationes inter abſciſſas & earum correſpondentes ordinatas exprimente iidem ſint; & ducantur duæ rectæ curvis in n punctis reſpective occurentes, tum contentum e ſingulis n partibus primæ rectæ inter vertices primæ curvæ,

curvæ, & punctum, ubi duæ rectæ lineæ ſeſe interſecant, erit ad contentum e ſingulis *(n)* partibus prædictæ primæ lineæ inter ejus vertices ſecundæ curvæ, & prædictum punctum poſitis, ut contentum ex *n* partibus adjacentibus ſecundæ rectæ lineæ inter ejus vertices primæ curvæ, & punctum ubi duæ rectæ lineæ ſeſe interſecant, ad contentum e ſingulis *(n)* partibus ſecundæ rectæ lineæ inter ejus vertices ſecundæ curvæ, & prædictum punctum.

Per vertices lineæ intelligo puncta, in quibus curvam ſecat linea.

Cor. 2. Sit curva *n* dimenſionum, cujus crura in infinitum pergentia *n* habent aſymptotos, & æquatio ad curvam & ejus crurum aſymptotos habet terminos *(n* & *n* — 1*)* dimenſionum eoſdem; & conſequenter omnis proprietas curvæ, quæ ex iis terminis pendet, erit etiam proprietas aſymptotûm.

Curvarum diametri pendent e terminis æquationis, in quibus dimenſiones abſciſſæ & ordinatæ inveniuntur maximæ *(n)* & proxime inferiores *(n* — 1*)*, & conſequenter diameter ad aſymptotos erit etiam diameter ad curvam.

Hinc etiam ſequitur eandem habere inclinationem in datam diametrum aſymptotûm ac curvæ ordinatas.

Sint etiam ordinatæ ad abſciſſam utcunque inclinatæ, & earum ſumma ad *n* aſymptotos æqualis erit ſummæ earum ad curvam; & exinde ſummæ ordinatarum partium inter curvæ crura & correſpondentes aſymptotos interceptarum ex utrâque parte diametri erunt inter ſe æquales.

Eadem etiam affirmari poſſint de quibuſcunque curvis, in quarum æquationibus relationem inter abſciſſas & earum correſpondentes ordinatas exprimentibus, termini *n* & *n* — 1 dimenſionum iidem inveniuntur.

Def. Si diameter ita ſecetur, ut ſumma partium inter punctum ſectionis & vertices ex uno latere æqualis ſit ſummæ partium inter prædictum punctum ſectionis & vertices ex altero latere; tum punctum ſectionis dici poteſt centrum.

Locus centrorum eſt curva, quam dico centralem curvam.

Duæ

Def. 2. Duæ diametri ſe ipſas interſecent, & ad ſe invicem conſtanter tendant, & ultimo coeant, tum locus punctorum hujuſmodi ultimarum interſectionum eſt curva, quam dico diametrialem curvam.

Cor. 3. Sit curva n dimenſionum (n) habens aſymptotos, & eandem diametrialem & centralem curvam habent aſymptoti & curva.

Diametri enim curvæ & aſymptotum ſunt eædem, & exinde eadem ſunt centra, & ultimæ diametrorum interſectiones: hæc vero centra & interſectiones conſtituunt reſpective centralem & diametrialem curvam, & conſequenter eædem ſunt centrales & diametriales curvæ ad datam curvam n dimenſionum & ejus n aſymptotos.

Cor. 4. Si curva non habet n rectas aſymptotos, tum forſan inveniri poſſunt æquationes ad aſymptotos & ellipſes, quarum æquationibus in ſeſe ductis, termini reſultantis æquationis in quibus dimenſiones abſciſſæ & ordinatæ inveniuntur maximæ (n) & proxime inferiores ($n-1$), iidem erunt ac datæ curvæ, & conſequenter eaſdem habent diametros, centralem & diametrialem curvam aſymptoti & ellipſes conjunctim ac data curva.

Et ſic deinceps.

Cor. 5. Sint duæ curvæ n dimenſionum, & iidem ſint termini n & $n-1$ dimenſionum in æquationibus ad utramque curvam, ordinatæ vero earum ſecent curvam in n punctis, tum differentia inter areas unius curvæ ex hoc vel illo abſciſſæ latere contentas æqualis erit differentiæ inter areas alterius curvæ ex hoc vel illo abſciſſæ latere contentas.

Cor. 6. Abſciſſa & ejus ordinatæ curvam interſecent in n punctis, & ratio quam habet contentum ſub n partibus abſciſſæ inter ordinatam & n vertices abſciſſæ interceptis, ad contentum ſub ſingulis n ordinatis pendet e terminis æquationis, in quibus inveniuntur maximæ & abſciſſæ & ordinatæ conjunctim dimenſiones; & conſequenter ſi curva habet n aſymptotos, & eadem linea eſt abſciſſa ad aſymptotos & curvam, & ad abſciſſam ordinatæ eandem habeant inclinationem, tum contentum ſub ſingulis n ordinatis curvæ erit ad contentum ſub n ordinatis aſymptotum in ratione contenti ſub n partibus inter ordi-

natam & n vertices abſciſſæ curvæ interceptis, ad contentum ſub n partibus abſciſſæ inter ordinatam & n vertices abſciſſæ aſymptotum interceptis.

Et ſic de aſymptotibus, ellipſibus, &c. conjunctim: i. e. aſymptotibus cujuſcunque generis vel rectilineis vel curvilineis.

Cor. 7. Sint termini æquationis, in quibus maximæ invenientur abſciſſæ & ordinatæ (x & y) dimenſiones, rectum angulum ſecum facientes, contentum $\overline{y^2+x^2} \times \overline{y^2+x^2} \times \overline{y^2+x^2} \times$ &c. tum contentum ſub n ordinatis ſemper erit æquale contento ſub n partibus abſciſſæ inter ordinatam & n vertices abſciſſæ interceptis; & omnes ordinatæ cum ſuis diametris rectum angulum faciunt.

Cor. 8. Sint termini æquationum ad duas curvas, in quibus inveniuntur maximæ n, $n-1$ & $n-2$ dimenſiones, iidem; tum ſumma n ſubnormalium, n ſolidorum e rotatione curvæ circa quamcunque abſciſſam generatorum, & quadratorum linearum e quibuſcunque datis punctis ductarum ad (n) puncta reſpective, in quibus eadem recta linea ſecat curvam, erit eadem in unâ curvâ ac in alterâ.

Cor. 9. Sint termini æquationis, in quibus inveniuntur maximæ dimenſiones $\overline{y^2+ax^2}^{n} + b \times \overline{y^2+ax^2}^{n-1}$, ubi x & y abſciſſam & ejus ordinatas reſpective denotant: tum ſumma quadratorum e ſingulis conjugatis diametris erit data quantitas.

Et ſic de innumeris hujuſmodi propoſitionibus.

PROB. XI.

Fig. 10. Sint deſcriptæ duæ curvæ, quarum æquationes relationem inter abſciſſas ($x=AP$, $z=a\Pi$) & earum ~~reſpectivas~~ correſpondentes ordinatas ($y=PM$, $v=\Pi M$) exprimentes, ſunt $Ay^{n}+By^{n-1}+Cy^{n-2}+Dy^{n-3}+$ &c. $=0$. $av^{m}+bv^{m-1}+cv^{m-2}+dv^{m-3}+$ &c. $=0$; in quibus A, B, C, D, &c. a, b, c, d, &c. reſpective denotant rationales abſciſſarum (x & z) functiones; invenire omnes æquationes, quas harum duarum curvarum ope conſtruere licet.

Ita

Ita transformentur hæ æquationes, ut lineæ ($Ip=X$, $\gamma\pi=Z$) fiant abſciſſæ & ($pM=Y$, $\pi M=V$) fiant ordinatæ. Scribantur pro lineâ AL (a), IL (b), pro ſinubus angulorum MpL, MQp & MPL reſpective s, c & r; & pro ſinu anguli pMQ quæ erit $s\sqrt{1-c^2}+c\sqrt{1-s^2}$, ($S$); & ſinu anguli PLp quæ erit $r\sqrt{1-c^2}+c\sqrt{1-r^2}$, ($C$). Tum in æquatione $Ay^n+By^{n-1}+Cy^{n-2}+Dy^{n-3}+\&c.=o$, pro x ſcribatur $\frac{cX-SY}{r}+\frac{cb}{r}-a$; & pro y, $\frac{CcX+\overline{rs-CS}Y}{rc}+\frac{Cb}{r}$.

Sic etiam transformari poteſt altera æquatio, ut ejus abſciſſa fiat ($\gamma\pi$) & ordinata (πM), quæ angulum facit cum ejus abſciſsâ $\gamma\pi$ æqualem angulo MpL, cujus ſinus eſt s.

Pro abſciſſis (X, Z) ſcribatur in utrâque reſultanti æquatione (t), & pro y & V ſcribatur w, exorientur duæ æquationes duas incognitas quantitates t & w habentes; ita reducantur hæ duæ æquationes in unam, ut exterminetur t, reſultat æquatio cujus radix eſt w, & quæ continet in ſe omnes æquationes, quæ harum curvarum ope conſtrui poſſunt.

Cor. 1. Sint duæ curvæ datæ, e prædictis conſtat æquationem generalem, quam earum ope conſtruere licet, non plures quam ſex incognitas quantitates, viz. AL, IL, $\angle MQp$, $\angle MpQ$, $\alpha\gamma$ & $\angle\pi\gamma\Pi$, habere, & conſequenter duarum datarum curvarum ope haud generaliter conſtrui poſſunt æquationes plures quam ſex dimenſiones habentes.

Cor. 2. Si vero duæ prædictæ curvæ haud ſint datæ, ſed quaſdam (m) incognitas coefficientes in ſe involvant earum æquationes; tum ex hoc problemate inveniri poſſunt æquationes, quas prædictarum curvarum ope conſtruere licet.

Cor. 3. Datâ & æquatione conſtruendâ & curvæ æquatione, cujus ope illam conſtruere licet, innumeræ aliæ curvæ inveniri poſſunt, quarum abſciſſæ & ordinatæ eædem ſunt ac abſciſſæ & ordinatæ datæ

curvæ, & ordinatæ ad earum interſectionum puncta pertinentes erunt datæ æquationis radices.

Hæ datæ æquationes in quaſlibet quantitates ducantur, & a ſeſe ipſis ſubducantur, reſultant quæſitæ æquationes.

Cor. 4. Sit data æquatio $y^n - py^{n-1} + qy^{n-2} - \&c. = o$, conſtruenda; aſſumatur æquatio $Ay^m + \overline{a + bx}\, y^{m-1} + \&c. = o$: in datâ æquatione pro quibuſlibet terminis ſubſtitue eorum valores e poſteriori æquatione deductos, reſultat æquatio ad alteram curvam, e quâ & aſſumptâ facile conſtat conſtructio datæ æquationis: e. g. ſit aſſumpta æquatio $y^m = ax$, & ſit $m \times r = n - s$; ſubſtitue in quibuſdam locis datæ æquationis $(y^n + by^{n-1} + \&c. = o)$, pro y^m ejus valor ax, & reſultet $a^r x^r y^s + b a^r x^r y^{s-1} + \&c. = o$, e duabus æquationibus $y^m = ax$ & $a^r x^r y^s + \&c. = o$, conſtrui poteſt data æquatio: ex. 2. ſit data æquatio $y^m - ay + b = o$; aſſumatur æquatio ad parabolam $y^m = Ax$, pro y^m ſubſtitue in datâ æquatione Ax, reſultat æquatio ad rectam lineam $Ax - ay + b = o$; unde rectæ lineæ & prædictæ parabolæ ope conſtrui poteſt data æquatio.

Ex. 3. 1. Sit æquatio $y^6 + qy^4 - ry^3 + sy^2 - ty + w = o$, in quâ deficit ſecundus terminus, aſſumatur æquatio ad cubicalem parabolam $y^3 = px$, & ſcribantur in data æquatione pro y^6, y^4 & y^3 reſpective p^2x^2, pxy & px, reſultat æquatio $p^2x^2 + qpxy - rpx + sy^2 - ty + w = o$ ad conicam ſectionem, cujus ope & cubicalis parabolæ conſtruere liceat datam æquationem.

2. Sit æquatio $y^5 + qy^3 - ry^2 + sy - t = o$; in hâc æquatione pro y^5, y^3 & y^2 ſcribantur reſpective $\frac{p^2x^2}{y}$, px & $\frac{px}{y}$, reſultat æquatio $p^2x^2 + qpxy - rpx + sy^2 - ty = o$ ad conicam ſectionem, cujus ope deſcribi poteſt data æquatio.

Cor. 5. Æquatio data conſtruenda, cujus radix eſt y, ſæpe habet poſſibiles radices, cum nulla datur poſſibilis interſectio, i. e. poſſibilis valor radicis vel ordinatæ y impoſſibilem habet valorem abſciſſæ x ſibi ipſi reſpondentem, & hoc fit ut conſtat e Medit. Algeb. quando duo

vel

vel plures abſciſſæ (x) valores uni valori ordinatæ y reſpondent; unde in prædictis aſſumptis æquationibus maxime cavendum eſt, ne cum y ſit poſſibilis x fiat impoſſibilis quantitas, & exinde e conſtructione haud inveniatur poſſibilis radix datæ æquationis; quod ſemper evitari poſſit, ſi modo aſſumatur æquatio ad parabolam $y^m = ax$, &c.

Cor. 6. Æquatio ($n \times m$) dimenſionum conſtrui poteſt duarum curvarum ope, quarum dimenſiones in ſeſe ductæ ad minimum æquant datæ æquationis dimenſiones ($n\,m$). e. g.

Æquationes ſeptem dimenſionum conſtrui poſſunt vel ope conicarum ſectionum & curvæ quatuor dimenſionum vel duarum curvarum tres dimenſiones habentium, &c.

In nonnullis caſibus conſtruere præſtat æquationem, cujus diviſor eſt data æquatio.

Cor. 7. Curva n dimenſionum duci poteſt per $\left(n \times \frac{n+3}{2}\right)$ puncta; tot enim incognitas & independentes coefficientes habere poteſt n dimenſionum æquatio $y^n + \overline{a+bx}\,y^{n-1} + \overline{c+dx+ex^2}\,y^{n-2} + \&c. = 0$, ſed $n.\frac{n+3}{2}$ puncta haud ſemper ſufficiant ad determinandam curvam n dimenſionum, forſan enim curva n dimenſionum duci poteſt per $n.\frac{n+3}{2}$ data puncta, in cujus æquatione haud continetur terminus y^n: ſint duæ curvæ n & m dimenſionum, & in curva m dimenſionum aſſumantur plura quam $n\,m$ puncta ſed haud plura quam $n \times \frac{n+3}{2}$, per quæ requiritur ducere curvam n dimenſionum, tum curva n dimenſionum reduci poteſt in duas vel plures, quarum una eſt altera curva m dimenſionum vel ejus nonnulla crura, i. e. duæ æquationes ad curvas communem habent diviſorem.

Cor. 8. Duæ curvæ $n \times m$ dimenſionum ſeſe interſecare poſſunt in $n \times m$ punctis; ſed datis $n \times m$ punctis minime generaliter ſequitur, quod dari poſſint duæ curvæ n & m dimenſionum, quæ ſeſe interſecare

care poſſint in quibuſcunque $n \times m$ datis punctis; ſit enim $m. \frac{m+3}{2}$ minor quam $n \times m$, per $m. \frac{m+3}{2}$ e prædictis $n \times m$ punctis duci poteſt curva m dimenſionum, ſed neceſſario ſequitur, quod omnia reliqua puncta $n \times m - m. \frac{m+3}{2}$ interſectionum in peripheria hujus curvæ exiſtunt.

Cor. 9. Sint $\sigma y^n + bxy^{n-1} + ex^2y^{n-2} +$ &c. & $\tau y^m + Bxy^{m-1} + Cx^2y^{m-2} +$ &c. termini, in quibus maximæ inveniuntur abſciſſarum & ordinatarum (x & y) ad duas curvas dimenſiones, & communem habeant diviſorem, tum haud aſcendit æquatio earum ope conſtructa ad nm dimenſiones, & ſic ſi duæ curvæ communem habeant aſymptoton, tum haud aſcendit æquatio earum ope conſtructa ad nm dimenſiones.

Cor. 10. Datis duabus æquationibus $py^n + \overline{a + bx}\, y^{n-1} + \overline{c + dx + ex^2}\, y^{n-2} +$ &c. $= 0$, & $Py^m + \overline{A + Bx}\, y^{m-1} + \overline{C + Dx + Ex^2}\, y^{m-2} +$ &c $= 0$, relationem inter abſciſſas & earum correſpondentes ordinatas duarum datarum curvarum exprimentibus; facile e curvis, quarum locos deſignant datæ æquationes, reduci poſſunt hæ duæ æquationes in unam, ita ut exterminetur incognita quantitas x.

Interſectiones enim harum curvarum erunt quæſitæ æquationis radices, datis æquationis radicibus facile cognoſcitur æquatio ipſa.

Cor. 11. Datâ æquatione $y^n - py^{n-1} + qy^{n-2} +$ &c. $= 0$, cujus radices ſint $\alpha, \beta, \gamma, \delta$, &c. transformetur hæc æquatio in alteram cujus radices ſint $\alpha + \beta$, $\alpha + \gamma$, $\beta + \gamma$, $\alpha + \delta$, &c. & æquationis reſultantis inveniatur conſtructio, ut ſi modo ordinata ſit poſſibilis quantitas, ejus abſciſſa etiam erit poſſibilis quantitas: e poſſibilibus radicibus æquationis reſultantis per conſtructionem inventis facile conſtant impoſſibiles datæ æquationis radices.

Cor. 12. Datâ æquatione duas incognitas quantitates (x & y) habente

bente, e conſtructione curvæ, cujus abſciſſa & ordinata ſint x & y, facile deduci poſſunt caſus, in quibus correſpondentes valores quantitatum (x & y) fiunt poſſibiles vel impoſſibiles. Non opus eſt hìc conſtruere æquationes; facile conſtant infinitæ hujuſmodi propoſitiones.

Cor. 13. Sint duæ algebraicæ curvæ, quarum æquationes ſint $Ay^m + bxy^{m-1} + \overline{d+ex^2}\,y^{m-2} + \&c. = 0$, & $Py^n + Qxy^{n-1} + \overline{R+Tx^2}\,y^{n-2} + \&c. = 0$, in quibus nulli continentur termini $m-1$ & $n-1$ dimenſionum reſpective, i. e. curvæ habeant idem generale centrum, ut conſtat e prob. 2. tum ſummæ negativorum valorum abſciſſarum & ordinatarum ad puncta interſectionum duarum curvarum pertinentium erunt reſpective æquales ſummis earum affirmativorum valorum, ni termini n dimenſionum in unâ & m dimenſionum in alterâ communem habeant diviſorem, vel aliquæ abſciſſæ vel ordinatæ evadant impoſſibiles.

PROB. XII.

Deſcribere curvas, quarum æquationes relationes inter abſciſſas & earum correſpondentes ordinatas exprimentes dantur.

Inveniatur methodus conſtruendi generaliter datas æquationes, & confit problema.

Cor. 1. Hinc omnes curvæ duarum, trium & quatuor dimenſionum deſcribi poſſunt: datur enim methodus in genere conſtruendi æquationes trium & quatuor dimenſionum, circuli & conicæ parabolæ ope; datur igitur methodus deſcribendi curvas trium vel quatuor dimenſionum.

Cor. 2. Plurimæ ſunt curvæ ſuperioris ordinis, quæ quarumcunque duarum datarum curvarum inferiorum ordinum ope haud generaliter deſcribi poſſunt.

Æquationes enim, quas conſtruere licet prædictarum duarum cur-

curvarum ope, per cor. 1. prob. 11. haud ſæpe includunt omnes, utcunque variatur valor abſciſſæ, curvarum ſuperioris ordinis æquationes.

Hinc ſequitur in deſcribendis curvis n dimenſionum haud raro opus eſſe quamplurimis inferiorum ordinum curvis.

Cor. 3. Si recta rotetur, ſemper tranſeuns per datum curvæ n dimenſionum punctum, tum dimenſiones æquationis conſtruendæ ſolummodo erunt $n-1$; ſi per duplex curvæ punctum tranſit, tum dimenſiones æquationis conſtruendæ erunt $n-2$; ſi per punctum triplex tranſit, dimenſiones prædictæ æquationis erunt $n-3$; &c.

Tot enim reliquas radices habet æquatio n dimenſionum.

Cor. 4. Sint x & y abſciſſa & ejus ordinata cujuſcunque curvæ, ſi abſciſſa incipit a puncto duplici, tum deſunt termini æquationis relationem inter abſciſſam, & ejus correſpondentes ordinatas exprimentis $\alpha x+\beta y+K$; ſi incipit a puncto triplici, tum deſunt termini $\gamma x^2+\delta xy+\varepsilon y^2+\alpha x+\beta y+K$; ſi incipit a puncto quadruplici, tum deſunt termini $\zeta x^3+nx^2y+\theta xy^2+iy^3+\gamma x^2+\delta xy+\varepsilon y^2+\alpha x+\beta y+K$; & ſic deinceps.

Cum enim abſciſſa x ſit $=0$, tum erunt tot radices 2, 3, 4, &c. ordinatæ y nihilo æquales, ut conſtat. e Medit. Algebr.

Cor. 5. Curvam n dimenſionum, cujus $n-1$ crura ſeſe interſecant in dato puncto A, deſcribere.

Fig. 20. Sit æquatio relationem inter abſciſſam $AP=x$ a puncto A incipientem & ejus ordinatas (PM, y) in eam ad rectum angulum inſiſtentes exprimens $Ay^n+\overline{a+bx}\,y^{n-1}+\overline{dx+ex^2}\,y^{n-2}+$ &c. $=0$; ita transformetur hæc æquatio, ut exprimat relationem inter abſciſſam Ap, z & ordinatam pM, v.

Sint q & s ſinus & coſinus anguli ApP, tum ſubſtituatur in datâ æquatione pro x, qz & pro y, $v+sz$; coefficientes terminorum (z^n, z^{n-1}) erunt reſpective $As^n+bs^{n-1}q+es^{n-2}q^2+$ &c. & $as^{n-1}+ds^{n-2}q+bs^{n-3}q^2+$ &c. & conſequenter, ſi in abſciſsâ Ap ſumatur

linea

linea $Am = -\frac{a s^{n-1} + d s^{n-2} q + b s^{n-3} q^2 + \&c}{A s^n + b s^{n-1} q + e s^{n-2} q^2 + \&c}$, erit m punctum in peripheriâ curvæ.

Hinc facile inveniri poteſt quilibet punctorum numerus, per quæ tranſit curva.

Hujus quantitatis valor; vel curvæ, e geometriæ principiis ſequitur, conſtructio.

Si curva n dimenſionum habet punctum, in quo $n-2$ crura ſeſe interſecant; tum inveniri poſſunt puncta, per quæ tranſit curva, quadraticæ æquationis ope; unde facile conſtrui poteſt curva. Et ſic deinceps.

Non opus eſt hìc deſcribere curvas motu datorum angulorum vel curvarum circa data puncta, &c. e generalibus principiis erui poſſunt hujuſmodi deſcriptiones: datâ methodo curvæ deſcriptionis erui poſſunt ejus dimenſiones; vel in multis caſibus e numero punctorum, in quibus a rectâ lineâ ſecari poteſt; vel e dimenſionibus æquationis relationem inter abſciſſam & ejus correſpondentes ordinatas relationem exprimentis. e. g. fig. 21. Sint n puncta (a, b, c, d, &c.) & ſit m punctum, a quo ad data puncta (a, b, c, &c.) ducuntur lineæ (am, bm, cm, &c.) & ſit ſumma linearum $am + bm + cm + dm +$ &c. data quantitas; locus punctorum m non poteſt eſſe curva plurium quam 2^n dimenſionum.

Fig. 22. Datis quibuſdam curvis, ex iis ſæpe deduci poſſunt aliæ: e punctis datæ curvæ quolibet modo deriventur alia puncta, & conſequenter e datâ curvâ ſequitur altera. e. g. Sit curva MNS, & a dato puncto C in tangentem curvæ ſemper demittatur perpendiculum CP, tum locus hujuſmodi punctorum (P) erit deducta curva.

Et ſic a dato puncto C ducatur perpendiculum in curvæ perpendiculum, & formatur alia curva; ſed de his ſatis.

De curvis per data puncta tranſeuntibus vel datas lineas tangentibus.

1. Sit curva data, tum tria puncta vel tres tangentes, &c. ſufficiunt ad determinandam ejus poſitionem.

2. Numerus tangentium vel punctorum, quæ ſufficiant ad determinandam curvam, pendet e numero incognitarum coefficientium, quas habet data æquatio relationem inter abſciſſam & ejus correſpondentes ordinatas exprimens. e. g. Si curva n dimenſionum generale habet centrum, quod ſit C; duci poteſt curva per $n \times \frac{n+1}{2}$ puncta; tot enim incognitas coefficientes habet data æquatio $y^n + bxy^{n-1} + \overline{c + dx^2}\, y^{n-2} + \&c. = 0$. Hic autem numerus punctorum, &c. haud ſemper ſufficiant ad determinandam curvam prædictam n dimenſionum, forſan enim curva n dimenſionum, quæ generale habet centrum, duci poteſt per $n.\frac{n+1}{2}$ data puncta, in cujus æquatione haud continetur terminus y^n.

Datâ æquationis formulâ relationem inter abſciſſam & ejus correſpondentes ordinatas exprimentis; invenire curvam, quæ tranſit per data puncta, tangit datas lineas, &c.

Fig. 24. Aſſumatur linea AP pro abſciſsâ, alia vero PM pro ordinatarum directione; pro ſingulo dato puncto datur abſciſſa & correſpondens ordinata, quibus pro ſuis valoribus in datâ æquatione ſubſtitutis, reſultant tot ſimplices æquationes, quot dantur puncta.

Ducatur linea (nm) parallela ordinatæ MP, ſecans tangentem EF in n; e datâ æquatione inveniatur ratio, quam habent fluxiones abſciſſæ & ordinatæ, quæ eſt $Fm : mn$: inveniatur etiam ſubtangens, cui æqualis eſt $x + AF$; ita transformentur omnes hæ æquationes reſultantes in unam, ut exterminentur omnes præter unam incognitam coefficientem; reſultat æquatio, cujus incognita quantitas eſt ea coefficiens.

Fortaſſe vero requirit problema, ut prius transformetur abſciſſa in alteram, quod fieri poteſt per prob. primum.

E prædictis æquationibus conſtant dimenſiones reſultantis æquationis.

PROB.

PROB. XIII.

Sit fluxio abſciſſæ $AP = x$ invariabilis, datâ algebraicâ æquatione relationem inter abſciſſam (x) & ejus correſpondentes ordinatas $PM = y$ exprimente, invenire ſummam ſingulorum valorum fluxionalis quantitatis, quæ ſit quantitatum (x & y) functio, cujus fluens inveniri poteſt.

Inveniatur fluens, & e primo capite Medit. Algebr. inveniatur ſumma ſingulorum valorum hujuſce fluentis, hujuſce ſummæ fluxio erit ſumma quæſita.

Fig. 67. Ex. 1. Datâ abſciſsâ $AP = x$, & æquatione relationem inter abſciſſam (AP) & ejus correſpondentes ordinatas ($PM = y$) exprimente $y^n - \overline{a + bx}\, y^{n-1} + \overline{c + dx + ex^2}\, y^{n-2} + \&c. = o$, & ductis ($n$) tangentibus ML, $M'L'$, $M''L''$, &c. invenire ſummam ſingularum (n) ordinatarum per ſuas reſpectivas ſubtangentes diviſarum, i. e. ſummam $\frac{PM}{PL} \mp \frac{PM'}{PL'} \pm \frac{PM''}{PL''} + \&c.$

Sint α, β, γ, &c. reſpective diverſi valores ordinatæ (y), erit $\frac{\dot{\alpha}}{\dot{x}} + \frac{\dot{\beta}}{\dot{x}} + \frac{\dot{\gamma}}{\dot{x}} + \&c. = \frac{PM}{PL} \mp \frac{PM'}{PL'} \pm \frac{PM''}{PL''} + \&c.$ ſed $\alpha + \beta + \gamma +$ &c. per vulgarem algebram $= a + bx$, & conſequenter $\frac{\dot{\alpha}}{\dot{x}} + \frac{\dot{\beta}}{\dot{x}} + \frac{\dot{\gamma}}{\dot{x}} + \&c. = \frac{b\dot{x}}{\dot{x}} = b$ datæ quantitati $= \frac{PM}{PL} \mp \frac{PM'}{PL'} \pm \frac{PM''}{PL''} + \&c.$

Fig. 67. Ex. 2. Datâ abſciſsâ x, & æquatione relationem inter abſciſſam (x) & ejus correſpondentes ordinatas (y) exprimente $y^n + \overline{a + bx}\, y^{n-1} + \overline{c + dx + ex^2} \times y^{n-2} + \&c. = o$, ſummam ſubnormalium invenire.

Supponantur diverſi ordinatæ valores reſpective α, β, γ, &c. conſequenter diverſæ ſubnormales PN, PN', PN'', &c. erunt reſpective

tive $\frac{\alpha\dot{\alpha}}{\dot{x}}, \frac{\beta\dot{\beta}}{\dot{x}}, \frac{\gamma\dot{\gamma}}{\dot{x}}$, &c. ſit $(\dot{x})$ invariabilis quantitas, harum quantitatum fluentes erunt $\frac{\alpha^2}{2\dot{x}}, \frac{\beta^2}{2\dot{x}}, \frac{\gamma^2}{2\dot{x}}$, &c. per problem. primum noſt. Meditat. Algebr. inveniri poteſt $\alpha^2 + \beta^2 + \gamma^2 + \&c. = \overline{a + bx}^2 - 2 \times \overline{c + dx + ex^2}$; conſequenter ſubnormalium ſumma quæſita erit $\frac{\alpha\dot{\alpha} + \beta\dot{\beta} + \gamma\dot{\gamma} + \&c.}{\dot{x}} = \frac{\overline{a + bx} \times b\dot{x} - \overline{d + 2ex}\,\dot{x}}{\dot{x}} = ab - d + \overline{b^2 - 2e} \times x$.

Cor. 1. Si ordinatæ ſecent curvam in (n) punctis, & bis ſumma harum ſubnormalium nihilo æqualis ſit, tum abſciſſa nihilo æqualis erit utcunque variatur: iterumque ſit æquatio ad curvam $(ax^m + bx^{m-1} + cx^{m-2} + \&c.)\,y^{n-m} + (ex^{m+1} + fx^m + \&c.)\,y^{n-m-1} + (hx^{m+2} + kx^{m+1} + \&c.)\,y^{n-m-2} + \&c. = 0$, ſi ejus ordinatæ (y) ſecent curvam in $(n - m)$ punctis, & ſummæ ad $3m + 2$ abſciſſas (x) prædictarum $(n - m)$ ſubnormalium ſint nihilo æquales; tum vero quælibet ſit abſciſſa (x), prædicta ſumma nihilo æqualis erit.

Hoc corroll. ex ſubſequente principio deduci poteſt: ſint $\alpha, \beta, \gamma, \delta$, &c. $(n + 1)$ diverſæ quantitates; ſcribantur hæ quantitates $\alpha, \beta, \gamma, \delta$, &c. pro x in quantitate n dimenſionum $px^n - qx^{n-1} + rx^{n-2} - sx^{n-3} \ldots\ldots + Px - Q$, ſi ſingula reſultans quantitas nihilo æqualis evadat, tum neceſſario erunt nihilo æquales coefficientes p, q, r, s, &c. ſolummodo enim n radices habet æquatio $px^n - qx^{n-1} + rx^{n-2} - \&c. = 0$ (n) dimenſionum.

Cor. 2. Si curva generale habeat centrum, & ejus abſciſſa per centrum tranſeat, tum ſummæ n ſubnormalium erunt reſpective abſciſſis a centro incipientibus proportionales; exinde ſi omnes diametri ſint inter ſe parallelæ, tum ſummæ (n) ſubnormalium ad diametrum ductarum erunt data quantitas.

Cor. 3. Si ordinatæ ſecent curvam in n punctis, & ducantur perpendicula ad curvam in n prædictis punctis, tum ſumma ſubnormalium

malium proportionalis erit diſtantiæ ordinatarum a dato puncto abſciſſæ.

Ex. 3. Datâ æquatione $y^n - Ay^{n-1} + By^{n-2} + Cy^{n-3} + \&c. = 0$, ubi A, B, C, &c. rationales abſciſſæ x functiones reſpective denotant, y vero ejus ordinatam : invenire ſummam e ſingulis valoribus ejus areæ, i. e. fluentem fluxionis $(A\dot{x})$.

Datâ abſciſsâ, ſint α, β, γ, δ, &c. diverſi ordinatæ y valores, & erit $\alpha + \beta + \gamma + \delta + \&c. = A$, & conſequenter $\alpha\dot{x} + \beta\dot{x} + \gamma\dot{x} + \delta\dot{x} + \&c. = A\dot{x}$, quarum fluentes ſunt areæ ipſæ, & earum ſumma æqualis eſt fluenti fluxionis $A\dot{x}$.

Cor. 1. Sit curva algebraica, & conſtat ſummam e ſingulis ejus areæ valoribus ſemper inveniri poſſe finitorum terminorum abſciſſæ vel ordinatæ ope, earum circularium arcuum & logarithmorum; ſumma enim prædicta fluens eſt fluxionis $A\dot{x}$, ſed æquatio in genere algebraicarum curvarum eſt $\overline{p + qx + rx^2 + sx^3 + \&c.} \times y^m - \overline{P + Qx + Rx^2 + \&c.} \times y^{m-1} + \&c. = 0$, & conſequenter $A = \frac{P + Qx + Rx^2 + \&c}{p + qx + rx^2 + \&c}$. unde $A\dot{x} = \frac{P + Qx + Rx^2 + \&c}{p + qx + rx^2 + \&c} \times \dot{x}$; ſed fluens hujuſce fluxionis inveniri poſſit ope finitorum terminorum abſciſſæ x ejus circularium arcuum & logarithmorum, ergo ſumma quæſita.

Cor. 2. Datâ abſciſsâ x; & n ſunt diverſi ordinatæ y valores, & conſequenter n ſunt diverſæ prædictæ areæ, unde æquatio, cujus radix (v) eſt area ipſa (ſi modo ea area inveniri poſſit finitis terminis abſciſſæ & ordinatæ) n & non plures habere poteſt dimenſiones.

Cor. 3. Nulla curva, quæ habet hyperbolicum crus conici generis generaliter quadrari poteſt.

Ita transformetur enim data æquatio, ut reſultet æquatio relationem inter abſciſſam, quæ ſit hyperbolici cruris aſymptotos & ejus correſpondentes ordinatas exprimens.

In fluxione arearum ſummæ $A\dot{x}$; A, quæ rationalis eſt functio abſciſſæ x, erit fractio, quæ continet in ſuo denominatore ſimplicem

diviſorem & haud plures ei æquales $x+\alpha$, unde conſtat fluentem fluxionis $A\dot{x}$ haud generaliter in finitis terminis abſciſſæ inveniri poſſe.

Si vero curva ſit continua, ex eâdem expreſſione vel æquatione, quæ generaliter quadrat unum curvæ crus, quadrabuntur etiam ſingula, &c. Et conſequenter ſi ſumma e ſingulis valoribus areæ haud inveniri poteſt, tum pari ratione ſequitur nullum curvæ crus quadrabile eſſe.

Ex. 4. Invenire ſummam ſolidorum a rotatione curvæ crurum circa axem ſuum generatorum.

Sit æquatio relationem inter abſciſſam x & ejus correſpondentes ordinatas y exprimens $y^n + Ay^{n-1} + By^{n-2} + \&c. = o$, ubi A, B, &c. denotant rationales abſciſſæ x functiones: ſint α, β, γ, δ, &c. diverſi ordinatæ y valores, & fluxio ſummæ ſolidorum quæſitæ eſt $3,14159$, &c. $\times\, \overline{\alpha^2 + \beta^2 + \gamma^2 + \&c.} \times \dot{x}$, ſed $\overline{\alpha^2 + \beta^2 + \gamma^2 + \&c.} = A^2 - 2B$; & conſequenter $3,14159$ &c. $\times\, \overline{A^2 - 2B} \times \dot{x}$ eſt fluxio ejuſdem ſummæ, & ejus fluens ſumma ſolidorum quæſita.

Huic exemplo conſimilia applicari poſſint corollaria ac præcedenti.

Ex. 5. Datâ abſciſsâ x, invenire ſummam e reciprocis ſubtangentibus.

Sit ultimus datæ æquationis terminus $px^n - qx^{n-1} + rx^{n-2} - sx^{n-3} + \&c.$ qui æqualis eſt contento $(\alpha \times \beta \times \gamma \times \delta \times \&c.)$ ſub ſingulis ordinatæ y diverſis valoribus i. e. $\alpha \times \beta \times \gamma \times \delta \times \&c. = px^n - qx^{n-1} + rx^{n-2} - \&c.$ Inveniantur fluxiones ex utrâque æquationis parte, & oritur æquatio $\dot{\alpha}\,\overline{\beta\gamma\delta, \&c.} + \dot{\beta}\,\overline{\alpha\gamma\delta, \&c.} + \dot{\gamma}\,\overline{\alpha\beta\delta, \&c.} + \dot{\delta}\,\alpha\beta\gamma, \&c. = npx^{n-1}\dot{x} - \overline{n-1}\,qx^{n-2}\dot{x} + \overline{n-2}\,rx^{n-3}\dot{x} - \overline{n-3}\,sx^{n-4}\dot{x} + \&c.$ dividatur prior æquationis pars per contentum $(\alpha\beta\gamma\delta, \&c.)$ poſterior vero per quantitatem $px^n - qx^{n-1} + rx^{n-2} + \&c.$ ei contento æqualem, & reſultat æquatio

$$\frac{\dot{\alpha}}{\alpha\dot{x}} + \frac{\dot{\beta}}{\beta\dot{x}} + \frac{\dot{\gamma}}{\gamma\dot{x}} + \frac{\dot{\delta}}{\delta\dot{x}} + \&c. = \frac{npx^{n-1} - \overline{n-1}\,qx^{n-2} + \overline{n-2}\,rx^{n-3} - \&c}{px - qx^{n-1} + rx^{n-2} - \&c}.$$

Illæ

Illæ autem quantitates $\frac{\dot{\alpha}}{\alpha\dot{x}}$, $\frac{\dot{\beta}}{\beta\dot{x}}$, $\frac{\dot{\gamma}}{\gamma\dot{x}}$, &c. ſunt reſpective reciprocæ ſubtangentes, & earum ſumma quæſita eſt $\frac{npx^{n-1} - \overline{n-1}\,qx^{n-2} + \overline{n-2}\,r.x^{n-3} - \&c}{px^{n} - qx^{n-1} + rx^{n-2} - \&c}$.

Fig. 16. 1. Cor. Occurrat recta quævis, per datum punctum P ducta, curvæ algebraicæ in tot punctis (A, B, C, &c.) quot ſunt ejus dimenſiones; rectæ (AK, BL, CM, &c.) curvam tangentes in his punctis ſecent abſciſſam, i. e. alteram rectam lineam (PLM) per idem datum punctum ductam in punctis (K, L, M, &c.) erit ſumma reciprocarum ſubtangentium $\frac{1}{PK} + \frac{1}{PL} + \frac{1}{PM} +$ &c. data quantitas.

Summa reciprocarum ſubtangentium omnino pendet ex ultimo datæ æquationis termino $px^{n} - qx^{n-1} + rx^{n-2} -$ &c. abſciſsâ igitur eâdem manente, mutatâ vero ordinatarum in eam inclinatione, e ſubſtitutione uſitatâ in prob. 1. cor. 1. conſtat haud mutari ultimum datæ æquationis terminum $px^{n} - qx^{n-1} + rx^{n-2} -$ &c. & conſequenter ſi ordinatæ, diverſas habentes ad abſciſſam inclinationes, per idem punctum (P) abſciſſæ tranſeant, i. e. habeant eoſdem quantitatis x valores, erunt ſummæ reciprocarum ſubtangentium inter ſe æquales.

Fig. 15. Ex. 6. Sit æquatio $y^{n} - Ay^{n-1} + By^{n-2} -$ &c. $= 0$, ubi A & B ſint rationales functiones abſciſſæ x, & y ordinata ad abſciſſam rectum angulum habens: ſit MQ chorda curvaturæ in directione PM, & QR perpendiculum in tangentem demiſſum, facile deduci poteſt $\frac{MQ}{QR^2} = \frac{-\ddot{y}}{2\dot{x}^2}$; ſed per vulgarem algebram, ſi modo α, β, γ, &c. ſint diverſi valores ordinatæ y, erit $\alpha + \beta + \gamma +$ &c. $= A$, & conſequenter $\frac{\ddot{\alpha}}{\dot{x}^2} + \frac{\ddot{\beta}}{\dot{x}^2} + \frac{\ddot{\gamma}}{\dot{x}^2} +$ &c. $= \frac{\ddot{A}}{\dot{x}^2}$, ergo ſumma quantitatum hujuſmodi $\frac{MQ}{QR^2}$ erit $\frac{-\ddot{A}}{2\dot{x}^2}$.

THEOR.

THEOR. X.

Nulla datur algebraica curva, quæ habet ovalem ſeſe in dato puncto haud interſecantem, quæ generaliter quadrari poteſt.

Inveniatur enim generalis expreſſio ad aream terminis abſciſſæ x, fiat hæc expreſſio vel area impoſſibilis, cum x fiat α vel π; & ovalis contineatur inter valores abſciſſæ α & π; inveniatur fluxio datæ expreſſionis, ſed methodus fluxiones inveniendi eadem eſt ac methodus inveniendi æquationes, quarum radices ſunt limites inter radices α & π datarum æquationum; ſi radices α & π datarum æquationum ſint poſſibiles, poſſibilis etiam erit radix inter eas poſita; exinde probari poteſt ovalem ſeſe neceſſario interſecare, ſi ejus area generaliter inveniri poſſit.

Cor. 1. Ovales ſeſe ſecantes in datis punctis nonnunquam generaliter quadrabiles eſſe docebit etiam ſubſequens exemplum. Sit æquatio curvæ $y = \overline{2a - 2x} \times \sqrt{2ax - x^2}$, curva dici poteſt ovalis ſeſe interſecans in uno puncto, vel duæ ovales ſeſe interſecantes, ſed nullum habet crus in infinitum pergens; generaliter quadrari poteſt hæc curva, & ejus area in genere invenitur $\frac{2}{3} \times \overline{2ax - x^2}^{1\frac{1}{2}}$.

Cor. 2. Hinc immediate ſequitur nullius algebraici ſolidi contentum, quod continet ovale ſolidum, vel cujus ſectio ſit ovalis curva, quæ haud ſeſe interſecat, generaliter inveniri poſſe; ſi enim ſolidi ſectiones haud generaliter quadrabiles ſunt, minime poteſt generaliter inveniri ſolidi ipſius contentum.

PROB. XIV.

Datâ æquatione relationem inter abſciſſam (x) & ejus ordinatas (y) exprimente, & fluxionali quantitate, quæ eſt functio abſciſſæ (x) & ordinatæ (y) & earum fluxionum, cujus fluens haud inveniri poſſit; invenire ſummam ſingulorum hujus functionis valorum, etiamque æquationem, cujus radix eſt ea fluxionalis quantitas.

E datâ æquatione inveniatur prima fluxio ordinatæ (y) & ſic ſecunda, tertia, &c. fluxiones ordinatæ, quibus valoribus in datâ fluxionali æquatione ſubſtitutis reſultat algebraica quantitas, cujus ſumma e ſingulis valoribus erui poteſt e noſt. medit. capit. 1mo. Et ſic de reliquis terminis æquationis quæſitæ e capit. prædicto deducendis.

Ex. 1. Datâ æquatione curvæ $y^2 - Ay + B = 0$, ubi A & B ſint quæcunque functiones abſciſſæ (x), invenire æquationem, cujus radices ſint ordinatæ (y) fluxiones per abſciſſæ fluxionem diviſæ, i. e. $\left(\frac{\dot{y}}{\dot{x}} = w.\right)$

Sint duæ ordinatæ (y) radices reſpective α & β, & ſumma requiſitæ æquationis radicum erit $\frac{\dot{\alpha}+\dot{\beta}}{\dot{x}} = \frac{\dot{A}}{\dot{x}}$, & rectangulum ſub binis valoribus contentum erit fractio $\frac{\dot{\alpha}\dot{\beta}}{\dot{x}^2} = \frac{1}{\dot{x}^2} \times \frac{\dot{A}\alpha - \dot{B}}{2\beta - A} \times \frac{\dot{A}\beta - \dot{B}}{2\alpha - A}$; cujus denominator eſt $\overline{4\alpha\beta - 2A \times \overline{\alpha+\beta} + A^2} \times \dot{x}^2 = \overline{4B - A^2} \times \dot{x}^2$; numerator vero $\dot{A}^2\alpha\beta - \dot{A}\dot{B}\,\overline{\alpha+\beta} + \dot{B}^2 = B\dot{A}^2 - A\dot{A}\dot{B} + \dot{B}^2$, & conſequenter æquatio quæſita eſt $w^2 - \frac{\dot{A}}{\dot{x}}w + \frac{B\dot{A}^2 - A\dot{A}\dot{B} + \dot{B}^2}{\overline{4B - A^2} \times \dot{x}^2} = 0.$

Ex. 2. Invenire æquationem, cujus radices ſint ſubtangentes curvæ, cujus æquatio eſt $y^2 - Ay + B = 0$.

Sint α & β duo ordinatæ (y) valores; & duæ ſubtangentes erunt $\frac{2\alpha^2 - A\alpha}{\dot{A}\alpha - \dot{B}} \times \dot{x}$ & $\frac{2\beta^2 - A\beta}{\dot{A}\beta - \dot{B}} \times \dot{x}$; quibus fractionibus ad communem denominatorem reductis, fit denominator $\dot{A}^2\alpha\beta - \dot{A}\dot{B} \times \overline{\alpha+\beta} + \dot{B}^2 = B\dot{A}^2 - A \times \dot{A}\dot{B} + \dot{B}^2$; numerator vero $\dot{x} \times \overline{2BA\dot{A} - 2BA\dot{A} - 2\overline{A^2 - 2B} \times \dot{B} + A^2\dot{B}}$; & conſequenter ſumma ſubtangentium erit $\frac{4B\dot{B} - A^2\dot{B}}{\dot{A}^2B - \dot{B}\dot{A}A + \dot{B}^2}\dot{x}$.

Fractionis, quæ ſit productum ſub duabus ſubtangentibus contentum, numerator erit $\dot{x}^2 \times \overline{4\alpha^2\beta^2 - 2A\alpha\beta \times \overline{\alpha+\beta} + A^2\alpha\beta} = \dot{x}^2 \times$

$\overline{4B^2 - 2A^2B + A^2B} = \overline{4B^2 - A^2B} \times \dot{x}^2$, & conſequenter æquatio, cujus radices ſunt ſubtangentes (w), erit $w^2 - \frac{\overline{4B\dot{B} - A^2\dot{B}} \times \dot{x}}{\dot{A}^2B - \dot{B}\dot{A}A + \dot{B}^2} w + \frac{\overline{4B^2 - A^2B} \times \dot{x}^2}{\dot{A}^2B - \dot{B}\dot{A}A + \dot{B}^2} = 0.$

Ex. 3. Datâ æquatione curvæ $y^3 + By - C = 0$, in quâ B & C ſunt quæcunque abſciſſæ (x) functiones, & y ſit ordinata; invenire æquationem, cujus radices ſint $\frac{\dot{y}}{\dot{x}} = Z$.

Sint tres radices ordinatæ (u, v, w,), tum ſumma earum fluxionum erit $\dot{u} + \dot{v} + \dot{w} = 0$. & $\frac{\dot{u}\dot{v} + \dot{u}\dot{w} + \dot{v}\dot{w}}{\dot{x}^2} = \frac{\dot{C} - \dot{B}u}{3u^2 + B} \times \frac{\dot{C} - \dot{B}v}{3v^2 + B} \times \frac{1}{\dot{x}^2} + \frac{\dot{C} - \dot{B}u}{3u^2 + B} \times \frac{\dot{C} - \dot{B}w}{3w^2 + B} \times \frac{1}{\dot{x}^2} + \frac{\dot{C} - \dot{B}w}{3w^2 + B} \times \frac{\dot{C} - \dot{B}v}{3v^2 + B} \times \frac{1}{\dot{x}^2}$; reducantur hæ fractiones in communem denominatorem, & erit denominator $\overline{3u^2 + B} \times \overline{3v^2 + B} \times \overline{3w^2 + B} = 27C^2 + 4B^3$; numerator vero $\overline{3w^2 + B} \times \overline{\dot{C} - \dot{B}u} \times \overline{\dot{C} - \dot{B}v} + \overline{3v^2 + B} \times \overline{\dot{C} - \dot{B}u} \times \overline{\dot{C} - \dot{B}w} + \overline{3u^2 + B} \times \overline{\dot{C} - \dot{B}v} \times \overline{\dot{C} - \dot{B}w} = \dot{B}^2B^2 - 3B\dot{C}^2 + 9C\dot{B}\dot{C}$; ergo ſumma rectangulorum ex quibuſque binis in ſeſe ductis, erit $\frac{\dot{B}^2 \times B^2 - 3\dot{C}^2B + 9C\dot{B}\dot{C}}{27C^2 + 4B^3}$; & contentum ſub tribus radicibus ($\dot{y}\,\dot{v}\,\dot{w}$) erit $\frac{\dot{C}^3 - \dot{B}^3C + \dot{B}^2\dot{C}B}{27C^2 + 4B^3}$; unde æquatio quæſita erit $Z^3 + \frac{\dot{B}^2B^2 - 3\dot{C}^2B + 9C\dot{B}\dot{C}}{\overline{27C^2 + 4B^3} \times \dot{x}^2} Z - \frac{\dot{C}^3 - \dot{B}^3C + \dot{B}^2\dot{C}B}{\overline{27C^2 + 4B^3} \times \dot{x}^3} = 0.$

Cor. 1. Eodem modo deduci poteſt æquatio, cujus radices ſunt ſecundæ datæ æquationis ordinatæ (y) fluxiones ex æquatione, cujus radices ſunt primæ datæ æquationis ordinatæ fluxiones, quo deducitur ea æquatio ex datâ, & ſic deinceps.

Ex. 4. Invenire ſubtangentium curvæ ſummam, cujus æquatio eſt $y^3 + By - C = 0$.

Sint

Sint (u, v, w,) diverſi ordinatæ (y) valores, & ſubtangentes erunt reſpective $\frac{\overline{3u^3+Bu}\times\dot{x}}{C'-B'u}$, $\frac{\overline{3v^3+Bv}\times\dot{x}}{C'-B'v}$, $\frac{\overline{3w^3+Bw}\times\dot{x}}{C'-B'w}$, quibus fractionibus ad communem denominatorem reductis, fit denominator $-B'^3uvw+B'^2C'\times\overline{uv+uw+vw}-B'C'^2\times\overline{u+v+w}+C'^3=C'^3-B'^3C+B'^2C'B$; & eâdem methodo invenitur numerator $\overline{4B^2C'B'+9CC'^2-3BCB'^2}\times\dot{x}$, & conſequenter ſumma ſubtangentium quæſita eſt $\frac{\overline{9C'^2C-3B'^2CB+4B'C'B^2}\times\dot{x}}{C'^3-B'^3C+B'^2C'B}$.

His principiis poſitis inveniri poſſunt infinitæ conſimiles curvarum proprietates; quales æquationes, quarum radices ſint radii curvaturæ, &c. vel quæcunque fluxionalis quantitas.

Hoc problema multis in caſibus reſolvi poteſt e transformatione duarum vel plurium æquationum in unam, ita ut incognitæ quantitates exterminentur.

THEOR. XI.

Fig. 16. Sint n rectæ (AB, CD, &c.) n etiam aliæ (EF, GH, &c.) n aliæ (LN, OP, &c.) & ſic deinceps; & a quocunque puncto M ad has rectas ducantur rectæ $M\alpha$, $M\beta$, &c. $M\varepsilon$, $M\zeta$, &c. $M\lambda$, $M\mu$, &c. in datis angulis vel directionibus, ſingulæ ad ſingulas; & ducantur contenta ſub n lineis $M\alpha\times M\beta\times$ &c. & $M\varepsilon\times M\zeta\times$ &c. & $M\lambda\times M\mu\times$ &c. reſpective in quaſlibet datas quantitates, quantitatum reſultantium ſumma nihilo æqualis fiat; & locus punctorum M vel erit curva n dimenſionum, vel erunt curvæ vel rectæ, quarum dimenſiones ſimul ſumptæ erunt n.

Aſſumatur enim recta Ap pro abſciſsâ (x), & pM pro ordinatâ (y), tum $M\alpha=ay+bx+c$, $M\varepsilon=dy+ex+f$, $M\lambda=ly+mx+n$,
$M\beta=a'y+b'x+c'$, $M\zeta=d'y+e'x+f'$, $M\mu=l'y+m'x+n'$,
&c. &c. &c. &c.

Ducantur hæ quantitates ($M\alpha$, $M\beta$, &c. & $M\varepsilon$, $M\zeta$, &c. & $M\lambda$, $M\mu$, &c. &c.) reſpective in ſeſe, &, quoniam ſingula quantitas unam

 ſolum-

ſolummodo habet dimenſionem abſciſſæ, vel ordinatæ y, contentum ſub n quantitatibus hujuſce generis (n) habet dimenſiones; ergo ſingula contenta reſultantia n habent dimenſiones; multiplicentur hæc contenta reſpective in datas quantitates, & fiat ſumma quantitatum reſultantium nihilo æqualis; & reſultat æquatio relationem inter abſciſſam $(Ap = x)$ & ejus correſpondentes ordinatas $(pM = y)$ exprimens, n dimenſiones habens.

Fig. 17. Ex. 1. Sit trapezium $(ABCD)$, & a puncto (M) ad ejus latera in datis angulis ducantur lineæ MQ, MR, MS, MT, ſingulæ ad ſingula; & ſit rectangulum linearum ductarum ad duo oppoſita latera $MQ \times MS$ in datâ ratione $(m : 1)$ ad rectangulum ductarum ad duo alia latera $MR \times MT$; tum punctum M vel erit punctum ad conicam ſectionem vel ad duas rectas lineas.

Aſſumatur enim pro abſciſsâ AQ (x) & pro (QM) ordinata y, tum

$$
\begin{array}{ll}
MS = ay + bx + c & \& \ MR = dy + ex + f \\
\times MQ = y & \quad \times MT = hy + kx \\
\hline
\end{array}
$$

$MS \times MQ = ay^2 + byx + cy = mdhy^2 + m\overline{he + kd}\,xy + mhfy + mekx^2 + mfkx$, quæ eſt æquatio ad conicam ſectionem vel duas rectas lineas.

Cor. 1. Hinc conſtat curvam tranſire per angulum trapezii (A), quoniam ſi abſciſſa (x) nihilo ſit æqualis, tum correſpondens ordinata (y) erit æqualis nihilo; & ſic probari poteſt conicam ſectionem tranſire per puncta $(B, C, D,)$ & conſequenter trapezium in conicâ ſectione inſcriptum eſſe.

Cor. 2. Sit conica ſectio, in quâ inſcribitur trapezium $(ABCD)$, & M quodlibet peripheriæ punctum; in lineas AB, BC, CD, DA in datis angulis ducantur lineæ MQ, MR, MS, MT; & erit rectangulum $MQ \times MS$ ad rectangulum $MR \times MT$ in datâ ratione.

Fig. 17. & 68. Cor. 3. Sit trapezium $ABCD$, & peripheriæ conicæ ſectionis punctum M, & ſit rectangulum $MQ \times MS$ ad rectangulum $MR \times MT :: r : s$; a puncto A ducantur lineæ AN & AP in datis angulis ad lineas BC & CD; & ſit $AN : AP :: a : b$: aſſumantur lineæ VO & LP in datis angulis ad lineas AB & AD; & ſit LP:

O

$QV :: MQ : MT :: ar : bs$; ducantur lineæ VR & PS parallelæ lineis EP & OV, & lineas AB & AD ſecantes in R & S, & ſeſe interſecantes in puncto H; erit HA tangens ad curvam in puncto A.

Fig. 18. Ex. 2. Sint tres lineæ AB, CD, EF; & tres aliæ BG, DH, FI; a puncto M in datis angulis ad lineas prædictas ducantur lineæ MQ, MR, MS, MV, MW, MZ reſpective; & ſit ſolidum $MQ \times MR \times MS$ ad ſolidum $MV \times MW \times MZ$ in datâ ratione; punctum M ponitur ad curvam ſecundi ordinis.

Hoc demonſtrari poteſt eodem modo, quo prius exemplum.

Fig. 18. Cor. 1. Puncta B, D, F, N, &c. in quibus lineæ AB, CD, EF ſecant lineas BG, DH, FI, erunt puncta ad curvam ſecundi ordinis prædictam.

Cor. 2. Sit curva ſecundi ordinis; & ducantur tres lineæ AB, CD, EF; & tres aliæ BG, DH, FI tres prædictas ſecantes in novem curvæ punctis (B, D, F, &c.) & a quolibet curvæ puncto in has lineas ducantur lineæ MQ, MR, MS, MV, MW, MZ reſpective in datis angulis; ſolidum $MQ \times MR \times MS$ erit ad ſolidum $MZ \times MV \times MW$ in datâ ratione.

Et idem affirmari poſſit in quibuſdam caſibus de lineis quarti, quinti, ſexti, &c. ordinum.

Fig. 19. Ex. 3. Sit curva ($m\,m'\,m''$ &c.) habens m dimenſiones, ſint etiam $n-m$ rectæ ($A'B$, CD, &c.) & n etiam rectæ (EF, GH, &c.) a quocunque puncto M in datâ directione (MP) ad curvam ducatur linea $Mm\,m'\,m''$ &c. eam ſecans in m punctis (m, m', m'', &c.) & ſic agantur in datis angulis ad rectas ($A'B$, CD, &c. EF, GH, &c.) lineæ $M\alpha$, $M\beta$, $M\gamma$, &c. $M\varepsilon$, $M\zeta$, Mn, &c. & ſit contentum ſub n lineis $Mm \times Mm' \times Mm'' \times$ &c. $M\alpha \times M\beta \times M\gamma \times$ &c. ad contentum ſub n lineis $M\varepsilon \times M\zeta \times Mn \times M\theta \times$ &c. in datâ ratione; tum punctum M erit punctum ad curvam n dimenſionum, vel ad curvas vel rectas, quarum dimenſiones ſimul ſumptæ ſunt n.

Aſſumatur enim recta (AP, x) pro abſciſsâ, & PM pro ordinatâ y; ſit etiam æquatio relationem inter abſciſſam ($AP=x$) & datæ curvæ ordinatas ($v=Pm$ vel Pm', &c.) ubi puncta m, m' & m'', &c. in eâdem lineâ cum puncto M jacent, exprimens $Av^m+\overline{a+bx}\,v^{m-1}+\overline{c+dx+ex^2}\,v^{m-2}+$ &c. $=0$; pro cujus radicibus (v) Pm, Pm', Pm'', &c. ſcribantur reſpective α, β, γ, &c. tum evadent $Mm=PM-Pm=y-\alpha$, $Mm'=y-\beta$

$-\beta$, $Mm''=y-\gamma$, &c. & exinde $Mm \times Mm' \times Mm'' \times$ &c. $=\overline{y-\alpha}\times\overline{y-\beta}$ $\times\overline{y-\gamma}\times$ &c. $=\frac{1}{A}\times(Ay^{m}+\overline{a+b}\,x\,y^{m-1}+\overline{c+dx+ex^{2}}\,y^{m-2}+$ &c.) lineæ vero $M\alpha$, $M\beta$, $M\gamma$, &c. ſint $M\alpha=hy+kx+l$, $M\beta=py+qx+r$, &c. & ſic de lineis $M\varepsilon$, $M\zeta$, Mn, &c. ubi literæ h, k, l, p, q, r, &c. invariabiles denotant quantitates; tum contentum ſub m lineis ($Mm \times Mm' \times Mm'' \times$ &c.) & $n-m$ lineis $M\alpha\times M\beta\times M\gamma\times$ &c. i. e. $\frac{1}{A}\times$ $\overline{Ay^{m}+\overline{a+b}\,x\,y^{m-1}+\overline{c+dx+ex^{2}}\,y^{m-2}+\text{\&c.}}\times\overline{hy+hk+l}\times\overline{py+qx+r}\times$ &c. erit in datâ ratione ad contentum ſub n lineis $M\varepsilon\times M\zeta\times Mn\times$ &c. cujus proportionis terminis extremis & mediis in ſeſe ductis, & eorum rectangulis inter ſe æqualibus eſſe ſuppoſitis, reſultat æquatio in quâ n & non plures inveniantur dimenſiones incognitarum quantitatum (x & y) abſciſſæ & ordinatæ quæſitæ.

Cor. 1. Tangentes duci poſſunt ad puncta interſectionum eodem prorſus modo, quo in exemplo primo: facile etiam in quodvis curvæ punctum duci poſſunt eodem fere modo tangentes.

Fig. 68. 2. Cor. 2. Sit $m=2$, & $n-m$ erit $n-2$: ducantur duæ rectæ lineæ curvam n dimenſionum in n punctis (a, b, c, d, &c. p, q, r, s, &c.) reſpective ſecantes; per ſingulum punctum (p, q, r, &c.) in quo altera ſecat curvam, ducantur n rectæ lineæ (ap, bq, cr, &c.) curvæ in $n-2$ diverſis punctis præter ea (a & p, b & q, c & r, &c.) reſpective occurrentes: per $\overline{n-2}\times\frac{n+1}{2}$ puncta e reliquis $n\times\overline{n-2}$, in quibus prædictæ rectæ occurrant datæ curvæ n dimenſionum, ducatur curva $n-2$ dimenſionum; & ducantur rectæ lineæ gh, gi, gk, &c. a quibuſcunque punctis (g) datæ curvæ n dimenſionum in datis angulis ad n rectas ap, bq, cr, &c. & rectæ $g\pi$, $g\varrho$, in datis angulis ad duas rectas $abcd$ &c. & pqr &c. & recta $g\sigma$ in datâ directione, i. e. datæ lineæ parallelâ, ad curvam $n-2$ dimenſionum, & ei occurrens in $n-2$ punctis (τ, υ, ϕ, &c.); erit contentum $gh\times gi\times gk\times$ &c. in datâ ratione ad contentum $g\pi\times g\varrho\times g\sigma\times g\tau\times g\upsilon\times$ &c.

Si enim $gh\times gi\times gk\times$ &c. ſit in datâ ratione ad contentum $g\pi\times g\varrho\times g\sigma\times g\tau\times$ &c. tum per ex. 3. locus punctorum g ſit curva n dimenſionum; hæc autem curva tranſit per omnia puncta, in quibus (n) rectæ ap, bq, cr, &c. occurrant curvæ prædictæ ($n-2$) dimenſionum, & conſequenter

ſequenter per $\overline{n-2} \times \frac{n+1}{2}$ puncta prædicta, etiamque per $(2n)$ puncta, in quibus (n) prædictæ rectæ ſecant duas rectas $a\,b\,c\,d$, &c. & $p\,q\,r\,s$, &c. & conſequenter tranſit per $n \times \frac{n+3}{2} - 1 = \overline{n-2} \times \frac{n+1}{2} + 2n$ puncta eadem ac prædicta curva (n) dimenſionum; at ratio contenti $gh \times gi \times gk \times$ &c. ad contentum $g\pi \times g\varrho \times g\sigma \times$ &c. in utrâque curvâ (n) dimenſionum eadem erit; conſequenter fere hæ duæ curvæ eædem evadent. Si duæ curvæ n dimenſionum tranſeunt per n^2 puncta eadem, & ratio contenti $gh \times gi \times gk \times$ &c. ad contentum $g\pi \times g\varrho \times g\sigma \times$ &c. in utrâque curvâ eadem ſit, tum neceſſario hæ duæ curvæ evadent eædem.

Sint plures curvæ, & a quocunque puncto in eas ducantur lineæ in datis directionibus; eadem erit ratiocinandi methodus de locis punctorum prædictis methodis deductorum.

Hinc facile deduci poſſunt multæ hujuſmodi proprietates, quæ omnibus algebraicis curvis competunt; e. g. ſint quæcunque lineæ, quæ exprimantur terminis abſciſſæ & ordinatæ (x & y) &c. e noſtr. Medit. Algebr. inveniri poteſt an data æquatio $Ay^n + \overline{a+bx}\,y^{n-1} + \overline{c+dx+ex^2}\,y^{n-2}$ + &c. $= 0$ eſt integra harum linearum functio: ex. 2. curvæ parium dimenſionum ſæpe formari poſſunt ex integris functionibus linearum in quæcunque data puncta ductarum; nullam enim habet irrationalitatem abſciſſæ & ejus ordinatæ quadratum lineæ a dato puncto ad curvam ductæ.

THEOR. XII.

1. Sint duæ æquationes $Ay^n + \overline{a+bx}\,y^{n-1} + \overline{c+dx+ex^2}\,y^{n-2} +$ &c. $= 0$, & $Py^m + \overline{p+qx}\,y^{m-1} + \overline{r+sx+tx^2}\,y^{m-2} +$ &c. $= 0$; reducantur hæ duæ æquationes in unam, ita ut exterminetur incognita quantitas (x); ſi reſultans æquatio, cujus radix eſt y, (nm) habet dimenſiones; tum ſumma ejus radicum ſolummodo pendet e terminis n & $n-1$ dimenſionum in unâ æquatione, m & $m-1$ dimenſionum in alterâ.

2. Sint duæ æquationes $Ay^n + \overline{a+bx}\,y^{n-1} + \overline{c+dx+ex^2}\,y^{n-2} +$ &c. $= 0$, & $Py^m + \overline{p+qx}\,y^{m-1} + \overline{r+sx+tx^2}\,y^{m-2} +$ &c. $= 0$; duæ vero aliæ æquationes eaſdem dimenſiones habentes $Av^n + \overline{\acute{a}+bz}\,v^{n-1} + \overline{C+dz+ez^2}\,v^{n-2} +$ &c. $= 0$, & $Pv^m + \overline{p+qz}\,v^{m-1} + \overline{R+sz+tz^2} \times v^{m-2} +$ &c. $= 0$. Sint termini n & $n-1$, m & $m-1$ dimenſionum in duabus præcedentibus æquationibus iidem ac termini n & $n-1$,

m & $m-1$ dimenſionum in duabus poſterioribus; & ſi mn valores habeant incognitæ quantitates x, y, z & v; tum ſummæ radicum quantitatum (x & y) æquales erunt ſummis radicum quantitatum (z & v) reſpective.

Conſtant hæc theorem. e cap. 4. Medit. Algebr.

Ex 1. Sit curva n dimenſionum, cujus diametri ſint omnes inter ſe parallelæ, & ſit abſciſſa diameter, dentur etiam n diverſæ ordinatæ, quarum ſubtangens ſit T; ſumma omnium prædictarum ordinatarum erit eadem quantitas, utcunque mutetur valor ſubtangentis T.

Si enim omnes diametri algebraicæ curvæ ſint parallelæ, tum per theorema 1. æquatio erit hujuſce formulæ $y^n + ay^{n-1} + \overline{c+dx}\,y^{n-2} +$ &c. $= 0$, & conſequenter ſubtangens $T = \dfrac{ny^n + \overline{n-1}\,ay^{n-1} + \overline{n-2}\,\overline{c+dx}\,y^{n-2} + \&c}{dy^{n-2} + \&c.}$; ergo termini, in quibus inveniuntur n & $n-1$ dimenſiones incognitarum quantitatum (x & y), iidem erunt in utrâque æquatione, utcunque mutetur valor ſubtangentis T; exinde conſtat exemplum.

Ex. 2. Sint duæ æquationes ad duas curvas, i. e. relationes inter abſciſſas (z & x) & earum correſpondentes ordinatas (y & v) duarum curvarum reſpective exprimentes $Ay^n + \overline{a+bx}\,y^{n-1} +$ &c. $= 0$, $Av^n \overline{a+bz}\,v^{n-1} +$ &c. $= 0$; & ſint termini n & $n-1$ dimenſionum in his duabus æquationibus iidem, & data ſubtangens T ad utramque curvam eadem; ſint etiam n^2 diverſæ ordinatæ vel abſciſſæ, quarum ſubtangens ad correſpondens curvæ punctum ſit T; tum ſumma hujuſce generis (n^2) ordinatarum vel abſciſſarum eadem erit in unâ curvâ ac in alterâ.

Subtangentes enim ad has duas curvas erunt reſpective

$$\frac{nAy^n + \overline{n-1}\,\overline{a+bx}\,y^{n-1} + \&c}{by^{n-1} + \&c} = T, \& \frac{nv^n + \overline{n-1}\,\overline{a+bz}\,v^{n-1} + \&c}{bv^{n-1} + \&c} = T,$$

unde reſultant quatuor æquationes $Ay^n + \overline{a+bx}\,y^{n-1} +$ &c. $= 0$, & $nAy^n + \overline{n-1}\,\overline{a+bz}\,y^{n-1} +$ &c. $- T(by^{n-1} +$ &c.$) = 0$; $Av^n + \overline{a+bz}\,v^{n-1} +$ &c. $= 0$, & $nAv^n + \overline{n-1}\,\overline{a+bz}\,v^{n-1} +$ &c. $- T(bv^{n-1} +$ &c.$) = 0$; at termini dimenſionum (n & $n-1$) inveniuntur iidem in duabus prioribus æquationibus ac in duabus poſterioribus; unde e caſ. 2. conſtabit exemplum.

Fig.

Fig. 69. Ex. 3. Sit curva n dimenſionum n habens aſymptotos (QB, RC, DS, &c.) a quolibet puncto P ad curvam ducantur n^2 perpendicula (Pa, Pb, Pc, &c.) etiamque a prædicto puncto P ducantur n perpendicula (PH, PI, PK, &c.) ad n aſymptotos; & curvæ ducantur ordinatæ (ab, bi, ck, &c.) ad datam abſciſſam AQ a prædictis curvæ punctis (a, b, c, &c.) His ordinatis ducantur lineæ $H\alpha$, $I\beta$, $K\gamma$, &c. parallelæ; erit ſumma $ab + bi + ck +$ &c. $= n \times \overline{H\alpha + I\beta + K\gamma + \text{\&c.}}$ & $Ah + Ai + Ak +$ &c. $= n \times \overline{A\alpha + A\beta + A\gamma + \text{\&c.}}$

Termini enim n & $n-1$ dimenſionum in æquationibus ad curvam & ejus aſymptotos iidem inveniuntur; etiamque termini, quorum dimenſiones ſunt maximæ vel proxime ſuperiores in æquationibus exprimentibus relationes inter abſciſſas & earum correſpondentes ordinatas ad puncta a, b, c, &c. in curvâ, & ad puncta H, I, K, &c. in aſymptotibus, iidem inveniuntur; (n) vero valores ſingularum linearum (PH, PI, PK, &c.) fingantur inter ſe æquales; exhinc per caſ. 2. facile conſtabit exemplum.

Idem affirmari poſſit de lineis ad curvam prædictam & ejus aſymptotos in quibuſcunque datis angulis ductis.

Ex. 3. Sint duæ æquationes ad duas curvas $Ay^n + \overline{a+bx}\,y^{n-1}$ + &c. $= 0$, & $A v^n + \overline{a+bz}\,v^{n-1} +$ &c. $= 0$, quarum abſciſſæ & earum correſpondentes ordinatæ æquales angulos ſecum reſpective faciunt, & ſint termini n & $n-1$ dimenſionum in duabus datis æquationibus iidem, ſupponantur abſciſſæ puncta P & p ad æquales diſtantias ab initiis abſciſſarum in reſpectivis curvis poſita; ſi a punctis P & p ducantur n^2 lineæ duas curvas reſpective tangentes in punctis a, b, c, &c. q, r, s, &c. & ex his punctis a, b, c, &c. q, r, s, &c. ducantur ordinatæ in reſpectivas datas abſciſſas; erunt ſummæ omnium harum ordinatarum vel abſciſſarum in duabus curvis reſpective inter ſe æquales.

Conſtant hæc & conſimilia ex hâc propoſitione.

THEOR. XIII.

Fig. 70. Sit æquatio $y = ax^n + bx^{n-1} + cx^{n-2} + dx^{n-3} +$ &c. relationem inter abſciſſam (x) & ejus correſpondentem ordinatam (y) curvæ $pqrst$ &c. exprimens.

Sint $n-1$ ordinatæ ad puncta p, q, r, s, t, &c. ad quæ tangentes reſpective parallelæ ſunt datæ abſciſſæ AP, ſecet vero curvam abſciſſa in

n

n punctis a, b, c, d, e, &c. erit contentum ſub lineis $Pp \times Qq \times Rr \times Ss \times$ &c. ad contentum ſub quadratis linearum $ab^2 \times ac^2 \times bc^2 \times ad^2 \times bd^2 \times cd^2 \times ae^2 \times be^2 \times ce^2 \times de^2 \times$ &c. $:: a^{n-2} : n^n$.

Sint duæ æquationes $y = ax^n + bx^{n-1} + cx^{n-2} +$ &c. & $nax^{n-1} + \overline{n-1}\, bx^{n-2} + \overline{n-2}\, cx^{n-3} +$ &c. $= 0$; exterminetur x, reſultat æquatio, cujus radix eſt y, viz. $y^{n-1} - Py^{n-2} + Qy^{n-3} \ldots Sy \pm T = 0$.

Sint $\alpha, \beta, \gamma, \delta$, &c. radices æquationis $ax^n + bx^{n-1} + cx^{n-2} + dx^{n-3} +$ &c. $= 0$, erit per Med. Algeb. $T : \overline{\alpha-\beta}^2 \times \overline{\alpha-\gamma}^2 \times \overline{\beta-\gamma}^2 \times \overline{\alpha-\delta}^2 \times \overline{\beta-\delta}^2 \times \overline{\gamma-\delta}^2 \times$ &c. $:: a^{n-1} : n^n$. Cum autem tangens parallela ſit abſciſſæ, tum fluxio ordinatæ (y) evaneſcet; inveniatur igitur fluxio datæ æquationis $y = ax^n + bx^{n-1} + cx^{n-2} +$ &c. & fiat fluxio $\dot{y} = 0$, reſultat æquatio $nax^{n-1} + \overline{n-1}\; bx^{n-2} + \overline{n-2}\; cx^{n-3} +$ &c. $= 0$, cujus radices erunt reſpective AP, AQ, AR, AS, &c. correſpondentes valores ordinatæ y. i. e. radices æquationis $y^n - Py^{n-1} + Qy^{n-2} \ldots Sy \pm T = 0$, erunt reſpective Pp, Qq, Rr, Ss, &c. & exinde invenietur $T = Pp \times Qq \times Rr \times Ss \times$ &c.

Radices $\alpha, \beta, \gamma, \delta$, &c. æquationis $ax^n + bx^{n-1} + cx^{n-2} + dx^{n-3} +$ &c. $= 0$, erunt reſpective abſciſſæ ad interſectiones datæ curvæ & abſciſſæ; i. e. erunt reſpective Aa, Ab, Ac, Ad, Ae, &c. & conſequenter $\alpha-\beta$, $\alpha-\gamma$, $\beta-\gamma$, $\alpha-\delta$, $\beta-\delta$, $\gamma-\delta$, &c. erunt reſpective $Ab - Aa$ (ab), $Aa - Ac$ $(-ac)$, $Ab - Ac$ $(-bc)$, $Aa - Ad$ $(-ad)$, $Ab - Ad$ $(-bd)$, $Ac - Ad$ $(-cd)$, &c. unde ſequitur $Pp \times Qq \times Rr \times Ss \times$ &c.$: ab^2 \times ac^2 \times bc^2 \times ad^2 \times bd^2 \times cd^2 \times$ &c. $:: a^{n-1} : n^n$.

PROB. XV.

Fig. 27. Datâ æquatione relationem inter abſciſſam & ejus correſpondentes ordinatas, rectum angulum cum abſciſsâ facientes, exprimente $Ay^n + \overline{a + bx}\, y^{n-1} + \overline{c + dx + ex^2}\, y^{n-2} +$ &c. $= 0$; circa abſciſſæ verticem P ducantur lineæ $(PA, PB, PD, PE$, &c) æquales angulos inter ſe facientes; invenire æquationem, cujus radices ſint reſpective lineæ $(PA, PB, PC$, &c.)

Sint l & r duorum arcuum (α & β) coſinus, quorum prior eſt multiplex m poſterioris, & radius eſt unitas; tum $r = \frac{1}{2}\sqrt[m]{l + \sqrt{l^2 - 1}} + \frac{1}{2}\sqrt[m]{l - \sqrt{l^2 - 1}}$. Et r erit reſpective coſinus arcuum $\frac{\alpha}{m}$, $\frac{360^\circ - \alpha}{m}$,

360°

$\frac{360^\circ+\alpha}{m}$, $\frac{720^\circ-\alpha}{m}$, & ſic deinceps, i. e. ſi modo peripheria circuli, cujus radius eſt unitas, dividatur in $2m$ æquales partes, quarum unum punctum diviſionis eſt arcus $\left(\frac{\alpha}{m}\right)$, tum erit r coſinus horum arcuum a ſe ipſis æqualiter diſtantium. Si vero r ſit coſinus, tum ſinus erit $\sqrt{1-r^2}$; ſit z radix æquationis quæſitæ, & abſciſſa x erit rz & ordinata $y=\sqrt{1-r^2}\,z$; quibus quantitatibus pro ſuis valoribus in datâ æquatione ſubſtitutis, reſultat æquatio, cujus radix eſt z.

Cor. 1. Exterminentur irrationales quantitates, & aſcendere poteſt æquatio reſultans ad $(2nm)$ dimenſiones ſi vero omnes poteſtates ordinatæ y ſint pares, tum deprimi poteſt æquatio in alteram nm dimenſiones habentem.

Cor. 2. Inventâ æquatione, cujus radix eſt z, invenitur ſumma radicum, rectangulorum e ſingulis duabus, contentorum ſub quibuſque tribus, &c. radicibus.

E principiis in noſt. Medit. Algeb. traditis ſine ſurdarum quantitatum exterminatione inveniri poteſt ſumma radicum, &c.

Facile eſt has proprietates in ellipſes circa centrum, &c. & alias curvas extendere: in præcedentibus propoſitionibus e datâ æquatione relationem inter abſciſſam & ejus ordinatas exprimente inventæ fuerunt proprietates; & vice versâ e datis proprietatibus inveniri poteſt formula æquationis.

Et ſic de innumeris hujuſmodi propoſitionibus.

Facile deduci poſſint proprietates curvarum, quæ correſpondent ſingulis propoſitionibus in noſtr. medit. algebr. contentis. Analytica enim problemata facile in geometrica transformari poſſint, & vice verſa geometrica in algebraica. e. g. Invenire utrum duæ æquationes $Ay^n+\overline{a+bx}\,y^{n-1}+\overline{c+dx+ex^2}\,y^{n-2}+\&c.=0$, & $Py^m+\overline{p+qx}\,y^{m-2}+\&c.=0$, communem habeant diviſorem; ſi ~~hæ~~ duæ curvæ his æquationibus designatæ conſtant e pluribus curvis quarum nonnullæ in ambabus æquationibus inveniuntur eædem, tum communem habent diviſorem datæ æquationes, ſin aliter vero non.

Et ſic de infinitis conſimilibus.

THEOR. XIV.

Fig. 71. Sint x, y, v abſciſſa & ordinatæ curvarum $ABCDEFGHI$, &c. & $A\beta\gamma\delta\varepsilon$ &c. & ſit $y = px^n$ & $v = \frac{n}{2.3} p a^{n-1} x - \frac{n.\overline{n-1}.\overline{n-2}}{30.2.3} p a^{n-3} x^3 + \frac{n.\overline{n-1}.\overline{n-2}.\overline{n-3}.\overline{n-4}}{42.2.3.4.5} p a^{n-5} x^5 - \frac{n.\overline{n-1}.\overline{n-2}.\overline{n-3}.\overline{n-4}.\overline{n-5}.\overline{n-6}}{30.2.3.4.5.6.7} p a^{n-7} x^7 + \frac{5n\,\overline{n-1}.\overline{n-2}.\overline{n-3}.\overline{n-4}.\overline{n-5}.\overline{n-6}.\overline{n-7}.\overline{n-8}}{66.2.3.4.5.6.7.8.9} p a^{n-9} x^9 - \frac{691.n.\overline{n-1}.\overline{n-2}.\overline{n-3}.\overline{n-4}.\overline{n-5}.\overline{n-6}.\overline{n-7}.\overline{n-8}.\overline{n-9}.\overline{n-10}}{2730.2.3.4.5.6.7.8.9.10.11} p a^{n-11} x^{11} +$ &c. in ultimo cujus termino dimenſiones abſciſſæ (x) debent eſſe $n-1$ vel $n-2$, prout (n) eſt par vel impar numerus.

Sit $z = AP = a$, biſecetur AP in Γ, & ducatur linea $E\Gamma\delta$, & ſi AE, EM, AM jungantur, erit triangulum $AEM = \Gamma P\varepsilon\delta\Gamma$ areæ.

Deinde biſecentur ΓP, $A\Gamma$ in R & V, & ducantur RG & $CV\gamma$, & jungantur AC, CE, EG, GM; & erunt duo triangula $ACE + ECM = V\Gamma\delta\gamma V$ areæ.

Eodem modo, ſi partes AV, $V\Gamma$, ΓR, RP iterum biſecentur in W, U, S, Q; & ducantur lineæ $BW\beta$, UD, SF, QH; & jungantur AB, BC, CD, DE, EF, FG, GH, HM; erunt quatuor triangula $ABC + CDE + EFG + GHM = WV\gamma\beta W$ areæ; & ſic deinceps.

Fig. 71. & 28. Cor. 1. Si curva ABC &c. M ſit conica parabola, i.e. $y = px^2$, erit $v = \frac{1}{3} pax$, & $A\beta\gamma\delta$, &c. erit recta linea, & hæc propoſitio eadem eſt cum celeberrimâ propoſitione Archimedis de quadraturâ parabolæ.

Cor. 2. Si $y = px^3$, erit $v = \frac{1}{2} pa^2 x$, & $A\beta\gamma\delta$ &c. iterum recta linea.

Cor.

Cor. 3. Datâ curvâ, cujus æquatio eſt $y = p x^{2n}$ vel $= p x^{2n+1}$; inveniri poteſt altera curva, cujus dimenſiones ſunt $(2n-1)$, in quâ ſummæ ejus ſegmentorum ad ſingulas biſectiones erunt reſpective æquales ſummis triangulorum datæ curvæ correſpondentium. Et vice versâ, data æquatione ad poſteriorem curvam, facile deduci poteſt æquatio ad priorem.

Hic adjici poteſt, quodſi loco biſectionis abſciſſa AP aliâ quâvis ratione in æquales partes dividatur; ſummæ triangulorum curvæ $ABCD$ &c. ad ſingulas diviſiones æquales erunt ſegmentis curvæ $A\beta\gamma\delta$ &c. correſpondentibus.

Lex ſeriei in hoc theoremate traditæ ita dici poteſt; ſit m par numerus, & (ſi $y = px^{m-2}$) ſit $v = \alpha x - \beta x^3 + \gamma x^5 - \delta x^7 + \varepsilon x^9 -$ &c. . . . . . . $\pm \lambda x^{m-5} \mp \mu x^{m-3}$; in hâc ſerie pro m in ſingulis terminis ſcribatur $m+2$ & pro a^{m-2} ſcribatur a^m, evadet ſeries reſultans $Ax - Bx^3 + Cx^5 - Dx^7 +$ &c. . . . . . . $\pm Lx^{m-5} \mp Mx^{m-3}$; tum ſi $y = px^m$, erit $v = Ax - Bx^3 + Cx^5 - Dx^7 \ldots\ldots \pm Lx^{m-5} \mp Mx^{m-3} \pm Nx^{m-1}$, ubi $\pm \frac{N}{m} = \frac{1}{2} - \frac{1}{m+1} - \frac{A}{2} + \frac{B}{4} - \frac{C}{6} + \frac{D}{8} \ldots \mp \frac{L}{m-4} \pm \frac{M}{m-2}$; coefficiens exinde termini x^{m-1} in ſerie prædictâ $v = \frac{n}{2 \cdot 3} pa^{n-1}x - \frac{n \cdot \overline{n-1} \cdot \overline{n-2}}{30 \cdot 2 \cdot 3} pa^{n-3}x^3 +$ &c. ubi $y = px^n$, erit $\pm \frac{N}{m} \times n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3} \cdot \frac{n-3}{4} \ldots \frac{n-m+3}{m-2} \cdot \frac{n-m+2}{m-1} \times pa^{n-m+1}$.

THEOR.

Fig. N. Invenire curvam, quæ tangat quaſcunque n datas lineas $a\alpha$, $b\beta$, $c\gamma$, $d\delta$, &c. datam abſciſſam $Apqrs$, &c. in punctis α, β, γ, δ, &c. ſecantes.

Ducantur a quibuſcunque punctis a, b, c, d, &c. in datis tangentibus lineæ ap, bq, cr, ds, &c. inter ſe parallelæ, & abſciſſæ $A\alpha p\beta q$, &c.

&c. aſſumptæ in punctis p, q, r, s, &c. occurentes; dicantur quotientes $\frac{ap}{\alpha p}, \frac{bq}{\beta q}, \frac{cr}{\gamma r}, \frac{ds}{\delta s}$, &c. reſpective $\pi, \varrho, \sigma, \tau$, &c. & pro $A\alpha, A\beta, A\gamma, A\delta$, &c. ſcribantur reſpective p, q, r, s, &c. ſit z in genere diſtantia inter datum punctum A & punctum, in quo tangens quæcunque occurrat prædictæ abſciſſæ $A\alpha p\beta q\gamma$, &c. Sint x & y abſciſſa & ejus correſpondens ordinata curvæ quæſitæ: aſſumantur $a + bz + cz^2 + dz^3 + \&c. = x$ abſciſſæ, & $A + Bz + Cz^2 + Dz^3 + \&c. = y$ ejus correſpondenti ordinatæ; unde $\frac{\dot{y}}{\dot{x}} = \frac{B + 2Cz + 3Dz^2 + \&c}{b + 2cz + 3dz^2 + \&c}$, exinde

$$\frac{B + 2Cp + 3Dp^2 + \&c}{b + 2cp + 3dp^2 + \&c} = \pi, \quad \frac{B + 2Cq + 3Dq^2 + \&c}{b + 2cq + 3dq^2 + \&c} = \varrho,$$

$$\frac{B + 2Cr + 3Dr^2 + \&c}{b + 2cr + 3dr^2 + \&c} = \sigma, \quad \frac{B + 2Cs + 3Ds^2 + \&c}{b + 2cs + 3ds^2 + \&c} = \tau,$$

&c. quibus æquationibus reductis, reſultent n ſimplices æquationes

$$\left.\begin{matrix} B + 2C \\ -\pi b - 2\pi c \end{matrix}\right) p + \left.\begin{matrix} 3D \\ -3\pi d \end{matrix}\right) p^2 + \&c. = o, \quad \left.\begin{matrix} B + 2C \\ -\varrho b - 2\varrho c \end{matrix}\right) q + \left.\begin{matrix} 3D \\ -3\varrho d \end{matrix}\right) q^2 + \&c. = o,$$

$$\left.\begin{matrix} B + 2C \\ -\sigma b - 2\sigma c \end{matrix}\right) r + \left.\begin{matrix} 3D \\ -3\sigma d \end{matrix}\right) r^2 + \&c. = o,$$

&c. $n + 1$ incognitas quantitates B, C, D, &c. b, c, d, &c. habentes; conſequenter ex aſſumptis quibuſcunque quantitatibus pro A, a & quâlibet unâ $(n+1)$ literarum prædictarum B, C, D, &c. b, c, d, &c. ſimplicium æquationum ope facile erui poſſunt reliquæ.

Cum vero requiratur, ut ſi modo z fiat vel p vel q, &c. tangens evadet aſſymptos; tum aſſumantur $\frac{A + Bz + Cz^2 + Dz^3 + \&c}{\overline{z-p} \cdot \overline{z-q} \cdot \&c} = x$, vel $\frac{a + bz + cz^2 + dz^3 + \&c}{\overline{z-p} \times \overline{z-q} \times \&c} = y$, &c. ſi vero (cum $z = p$) exigat problema, ut aſymptos evadet ordinatis y parallela; tum aſſumantur $\frac{A + Bz + Cz^2 + \&c}{\overline{z-p} \cdot \overline{z-q} \cdot \&c} = y$, & $\frac{a + bz + cz^2 + \&c}{\overline{z-q} \times \&c} = x$; in hoc enim caſu cum $x = z = p$; tum evadet y, haud vero x infinita quantitas.

Et

Et ſic aſſumi poſſint formularum haud multum diſſimilium quantitates pro abſciſsâ x & ejus correſpondente ordinatâ y curvæ, quæ plures habet datas aſymptotos.

Si vero formulæ crurum hyperbolicorum ad aſymptotos dantur, tum e datis formulis facile conſtabunt denominatores quantitatum, quæ pro abſciſsâ x & ejus ordinatis y aſſumi poſſunt. 2. Si requiratur curva, quæ tangat n datas tangentes in datis punctis & per m alia puncta data tranſeat: aſſumatur æquatio $a+bx+cx^2+dx^3 \ldots x^{2n+m-1}=y$, ubi x ſit abſciſſa, y vero ejus correſpondens ordinata; tum in hâc æquatione pro x & y ſcribantur earum $n+m$ dati correſpondentes valores, ad data puncta dantur abſciſſæ & earum correſpondentes ordinatæ, etiamque in æquatione $\frac{a+bx+cx^2+dx^3+\&c}{b+2cx+3dx^2+4ex^3+\&c}=v$, ubi v erit ſubtangens datæ curvæ, pro x & v ſcribantur n dati correſpondentes valores quantitatum x & v; reſultant $2n+m$ ſimplices æquationes tot idem incognitas coefficientes $a, b, c, d,$ &c. habentes, e quibus innoteſcent coefficientes ipſæ.

Si vero requiratur, ut curva habeat datas aſymptotos; facile e principiis prius traditis conſtabit formula maxime ſimplex æquationis quæſitæ relationem inter abſciſſam & ejus correſpondentes ordinatas exprimentis.

CAP. II.

De Curvoidibus & Epicurvoidibus ſive Curvis a Rotatione Curvarum ſuper rectas Lineas vel Curvas generatis.

Prob: XVI.

Fig. 32. CONCESSIS curvilinearum quadraturis, rectificare curvoidem ſeu curvam a puncto in curvâ ſuper rectam rotante generatam.

Rotetur curva MPL ſuper rectam lineam HG, ſit M punctum quod deſcribit curvam, cujus rectificatio requiritur; ducatur PI tangens curvam in puncto P, MI perpendiculum ad tangentem PI a dato puncto M; capiatur Lm perpendiculum in axem $= MI$, mr axi parallela $= PI$, &, ſi modo producatur, ſecans curvoidem in puncto n; agatur linea Lr, cui ſit parallela linea Qn; erit Qn perpendicularis ad curvam in puncto (n).

Ex hypotheſi $Lm = MI$, $mr = IP$, & $\angle Lmr = \angle MIP$, & conſequenter $Lr = MP$, & quando tangit axim in puncto (Q) rotans curva, n erit punctum curvoidem deſcribens, unde linea Qn erit perpendicularis ad curvoidem; conſequenter e ſimilibus triangulis Lmr & nst ſequitur curvoidis arcûs incrementum $nt = \frac{MP \times M^{\cdot}I\,(st)}{PI}$, cujus fluens eſt curvoidis arcus.

Fig. 33. Ex. 1. Sit curva MPL rotans, conici generis parabola, cujus æquatio relationem exprimens inter abſciſſam x ad axem & ejus ordinatas y ſit $px = y^2$, & punctum M curvoidem deſcribens ejus principalis vertex; tum erit $MP = \sqrt{y^2 + x^2} = \sqrt{px + x^2}$, $MI = \frac{\sqrt{p}\,x}{\sqrt{p + 4x}}$, $PI = \frac{\overline{p + 2x} \times \sqrt{x}}{\sqrt{p + 4x}}$, & $\frac{MP \times M^{\cdot}I}{PI} = \frac{\sqrt{p} \times \sqrt{p + x} \times \dot{x}}{p + 4x}$; cujus fluens & ejus conſtructio innoteſcunt.

Ex. 2. Sit curva (MPL) rotans conici generis ellipſis, & punctum M

M ellipſoidem deſcribens, ejus principalis vertex. Pro ellipſeos majori axe; ejus parametro; abſciſsâ, a puncto ellipſoidem deſcribente, incipiente; & ejus ordinatâ; ſcribantur reſpective $2a$, p, x & y: erunt $MP = \sqrt{x^2 + y^2} = \frac{\sqrt{2pax - \overline{p-2a}\,x^2}}{\sqrt{2a}}$, $MI = \frac{p^{\frac{1}{2}}ax}{\sqrt{2a \times \overline{2ax - x^2} + p \times \overline{a-x}^2}}$, & $PI = \sqrt{MP^2 - MI^2}$. Hinc inveniri poteſt ellipſoidis arcûs incrementum $\frac{MP \times \dot{M}I}{PI}$, & conſequenter per curvarum quadraturas ejus fluens, i. e. ellipſoidis arcus.

Fig. 32. Cor. 1. Iiſdem poſitis, invenire curvoidis aream; linea $mn = mr + rn$ (LQ vel arcui LP), & incrementum areæ curvoidis eſt $MI \times$ flux. $mn = MI \times \overline{\text{flux. } PI + \text{flux. arcus } LP}$; cujus fluens eſt area ipſa. e. g.

Sit curva rotans parabola, & punctum generans ejus principalis vertex, e prædictis conſtat $MI = \frac{\sqrt{p}\,x}{\sqrt{p+4x}}$ & $PI = \frac{\overline{p+2x}\sqrt{x}}{\sqrt{p+4x}}$, cujus fluxio eſt $\frac{8x^2 + 3px + \frac{1}{2}p^2}{\overline{p+4x}^{1\frac{1}{2}} \times x^{\frac{1}{2}}} \times \dot{x}$, & fluxio arcûs eſt $\frac{\sqrt{p+4x}\,\dot{x}}{2\sqrt{x}}$; ducantur hæ fluxiones reſpective in $MI = \frac{\sqrt{p}\,x}{\sqrt{p+4x}}$; reſultant $MI \times$ flux. $PI = \sqrt{p} \times \frac{8x^3 + 3px^2 + \frac{1}{2}p^2x}{\overline{p+4x}^2 \times x^{\frac{1}{2}}}\,\dot{x}$, & $MI \times$ flux. arcûs $LP = \frac{1}{2}\sqrt{px}\,\dot{x}$; & conſequenter $MI \times \overline{\text{flux. } PI + \text{flux. arcûs } LP} = \sqrt{p} \times \overline{\frac{1}{2}\sqrt{x} + \frac{8x^3 + 3px^2 + \frac{1}{2}p^2x}{\overline{p+4x}^2 \times x^{\frac{1}{2}}}} \times \dot{x}$, cujus fluens innoteſcit, & exinde area quæſita.

Cor. 2. Invenire curvoidis in quocunque puncto (n) curvaturæ radium, 1mo. Sint $\dot{z}$, $\dot{x}$, $\dot{y}$ reſpective $\frac{MP \times \dot{M}I}{PI}$, $\dot{M}I$ & $\frac{MI \times \dot{M}I}{PI}$,

e

e principiis bene notis radius curvaturæ in puncto n erit $\frac{\dot{z}^3}{\dot{y}\ddot{x} - \dot{x}\ddot{y}}$:

aliter; ſcribatur pro chordâ curvaturæ curvæ generantis in puncto P & directione MP, R; tum erit $R - 2Qn$ $(2MP) : 2Qn :: Qn :$ radium quæſitum.

Cor. 3. Datâ curvâ, cujus rotatione ſuper rectam lineam generatur curvois; invenire curvam, quæ ſit curvoidis involuta: radius (r) curvaturæ in puncto (n) fuit per præcedens corollarium $= \frac{2 \times Qn^2}{R - 2Qn}$; & quoniam $MP = Qn$, erit $r = \frac{2MP^2}{R - 2MP}$; unde ſi abſciſſa curvæ involutæ ſit $= Arc: MP \pm \frac{R \times PI}{R - 2MP}$, ejus correſpondens ordinata erit $\frac{R \times MI}{R - 2MP}$.

Cor. 4. Sit curva generans algebraica curva, ejus curvois etiam erit algebraica curva, vel non; prout curva generans rectificari poſſit in algebraicis terminis abſciſſæ & ordinatæ, vel non. In genere, utrum curva generans ſit algebraica, vel non; ejus curvois erit algebraica curva, vel non, prout ſumma $Arc: MP \pm PI$ exprimi poteſt in algebraicis terminis lineæ MI, vel non.

Cor. 5. 1. Datâ curvâ revolvente; invenire, utrum ſint aliquæ areæ ejus curvoidis quadrabiles; vel aliqui arcus, qui rectificari poſſint.

Inveniatur (ſi modo poſſibile ſit) area prædicta vel arcus generaliter in algebraicis terminis abſciſſæ & ordinatæ, & earum functionum circularium arcuum & logarithmorum, &c. Fiant illi termini, ſi modo fieri poſſint, qui involvunt circulares arcus & logarithmos, &c. nihilo reſpective æquales; reſultant areæ vel arcus, quæ quadrari, vel qui rectificari poſſunt.

2. Eodem fere modo inveniri poſſunt areæ vel arcus, qui datam inter ſe habent relationem.

Inveniantur enim generaliter duæ areæ, vel duo arcus, in finitis terminis abſciſſæ & ordinatæ, & earum functionum circularium arcuum

&

& logarithmorum, &c. Cogantur illi termini finiti, circulares arcus & logarithmi habere reſpective inter ſe datam, ſi modo talem admittant, relationem, & conſit corollarium. Si areæ vel arcus haud inveniri poſſint finitorum terminorum, circularium arcuum & logarithmorum, &c. ope; tum ſæpe inveniri poſſint ope convergentium ſerierum, & e principiis infinitarum ſerierum erui poſſit corollarii ſolutio.

Ex hiſce & fluxionum principiis inveſtigari poſſint ſolida generata a rotatione curvoidum circa axes ſuos, earum evolutæ curvæ, centra gravitatis curvoidum & prædictorum ſolidorum, & in genere quæcunque fluxionalis quantitas e terminis abſciſſarum & ordinatarum & earum fluxionum compoſita, vel ejus fluens: ſi enim ſubſtituantur pro abſciſsâ & ejus correſpondentibus ordinatis reſpective MI & $Arc:MP \pm PI$, & pro fluxionibus abſciſſæ & ordinatarum fluxiones quantitatum MI & $Arc:MP \pm PI$ reſpective; & ſic deinceps; exinde deducetur fluxionalis quantitas quæſita: ſi vero ejus fluens quæratur, e principiis fluxionum inveſtiganda eſt.

PROB. XVII.

Fig. 35. Iiſdem poſitis, rectificare epicurvoidem, i. e. curvam generatam a puncto in unâ curvâ ſuper alteram tanquam ejus baſim rotante.

1. Sit baſis curva (LAS), & curva revolvens LQP, rectificentur hæ curvæ inter ſe, & inveniatur relatio inter lineas, LP chordam curvæ revolventis, PQ perpendiculum a puncto P in tangentem LQR & LQ tangentem; LS ordinatam baſis ad abſciſſam perpendicularem, SR ejus ſubtangentem, & LR ejus tangentem; pro abſciſsâ AS ſcribatur x, & pro $x - \sqrt{LP^2 - \frac{\overline{PQ \times RS - SL \times LQ}^2}{LR^2}} = AS - LV$ (ſi modo LV ſit parallela abſciſſæ AS, & PV ad LV perpendicularis) ſcribatur z; epicurvoidis arcûs fluxio erit $\frac{LP \times \dot{z}}{PV} = \frac{LP \times LR \times \dot{z}}{PQ \times RS - SL \times LQ}$, cujus fluens eſt epicurvoidis arcus.

2. Sit

2. Sit $\dot{v}$ incrementum quantitatis $LS + \frac{PQ \times RS - SL \times LQ}{LR}$,
fluxio epicurvoidis arcûs erit $\frac{LP \times LR \times \dot{v}}{\sqrt{LP^2 \times LR^2 - \overline{PQ \times RS - LS \times LQ}^2}}$.

Fig. 36. 3. Datis ad ſingulum punctum P radiis curvaturæ, & curvæ revolventis, & curvæ ſuper quam tanquam baſim progreditur ea curva.

Ducantur circuli curvaturæ (LM & GH,) quorum radii ſint reſpective PD & PC; ſit Q punctum, quod deſcribit epicurvoidem, ducantur etiam lineæ CQ, PS perpendiculum a puncto contactûs P in lineam CQ, & DQ; cum radio DQ deſcribatur circulus ($QmNT$), ducantur linea Pr per punctum P parallela lineæ DQ, & occurrens lineæ CQ in r; linea Qv rectum angulum cum lineâ PD faciens; & Qn, Qm & Qo cotemporanea incrementa epicurvoidis, arcûs circuli ($QmNT$), & lineæ QC.

Facile conſtat lineam Qn eſſe ad rectum angulum ad lineam PQ, & lineam on ad lineam CQ; & exinde duo triangula Qon & PQS eſſe ſimilia, & conſequenter epicurvoidis incrementum $Qn = \frac{PQ \times Qo}{PS}$.

Epicurvoidis arcûs incrementum (Qn) etiam erit $\frac{Qm \times QP}{Pr}$; vel. etiam (ſi modo TN ſit diameter circuli $TQMn$) ſcribendo pro lineis Pr & Qm earum valores $\frac{DQ \times CP}{CD}$ & $\frac{TN \times \text{flux.}\, TQ}{\sqrt{TN^2 - TQ^2}}$ evadet $Qn =$
$\frac{TN \times CD \times PQ \times \text{flux.}\, TQ}{\sqrt{TN^2 - TQ^2} \times DQ \times CP} = \frac{2CD \times QP \times \text{flux.}\, TQ}{\sqrt{TN^2 - TQ^2} \times CP}$.

Ex. 1. Invenire epicycloidis longitudinem.

In poſtremâ formulâ incrementum arcûs epicycloidis fuit $Qn =$ $\frac{2CD \times QP \times \dot{T}Q}{\sqrt{TN^2 - TQ^2} \times CP}$, ſed in hoc caſu $\sqrt{TN^2 - TQ^2} = NQ = QP$, & conſequenter $Qn = \frac{2CD \times \dot{T}Q}{CP}$: CD & PC ſunt in caſu propoſito datæ quantitates, unde incrementum arcûs epicycloidis eſt in datâ

ratione

ratione ad incrementum lineæ QT, horum incrementorum inveniantur & corrigantur integrales, invenitur epicycloidis arcûs longitudo.

Ex. 2. Invenire curtatæ vel prolatæ epicycloidis arcûs longitudinem.

Scribantur pro lineis ND, NP, PC, NV reſpective a, b, c, x; unde in formulâ $\frac{2CD \times PQ \times \dot{TQ}}{\sqrt{TN^2 - TQ^2} \times CP}$, linea $PQ = \sqrt{\overline{b+x}^2 (PV^2) + \overline{2ax - x^2} (QV^2)} = \sqrt{b^2 + 2 \times \overline{b+a} \times x}$, & quantitates $\sqrt{TN^2 - TQ^2} = QN = \sqrt{2ND \times NV} = \sqrt{2ax}$ & $TQ = \sqrt{TV \times TN} = \sqrt{2a \times \overline{2a-x}}$, cujus fluxio $\dot{TQ} = \sqrt{2a} \times \frac{-\frac{1}{2}\dot{x}}{\sqrt{2a-x}}$; quibus quantitatibus pro ſuis valoribus in prædictâ formulâ ſubſtitutis, reſultat fluxio prolatæ vel curtatæ epicycloidis arcûs $\frac{\overline{a+b+c} \times \sqrt{b^2 + 2 \times \overline{b+a} \times x} \times \dot{x}}{c\sqrt{2ax - x^2}}$.

Cor. 1. Epicurvoidis curvaturæ radius vel inveniri poteſt e duabus primis formulis, e quibus inveniri poſſunt ejus abſciſſæ & ordinatæ fluxiones; vel ſi S & R ſint reſpective radii curvaturæ & curvæ revolventis & baſis ad punctum contactûs, i. e. S & R ſint PD & CP, & $D = PQ$ diſtantiæ puncti curvam generantis a puncto contactûs, & $C =$ chordæ curvaturæ curvæ rotantis LM in directione PQ; tum radius curvaturæ epicurvoidis in puncto Q quæſitus erit $\frac{2 \times \overline{R+S} \times D^2}{2 \times \overline{R+S} \times D - CR}$.

Hinc, cum $2 \times R + S \times D = CR$; tum radius curvaturæ erit infinitus, & ſæpe erit punctum contrariæ flexuræ.

Fig. 36. Cor. 2. Invenire involutam epicurvoidis; ſit epicurvois Qn, & baſis, ſuper quam rotetur curva epicurvoidem generans GPH, cum vero punctum deſcribens epicurvoidem poſitum ſit ad Q, tangat curva rotans baſim ad P, ducatur linea QPE, & (ſi R & S reſpective denotent radios curvaturæ baſeos & curvæ rotantis ad punctum P) pro chorda curvaturæ curvæ rotantis ad punctum P & in directione QP, ſcribatur C; pro diſtantiâ QP ſcribatur D; per coroll. 1. radius curvaturæ erit $\frac{2 \times \overline{R+S} \times D^2}{2 \times \overline{R+S} \times D - CR} = QE$, unde

$PE = QE - PQ(D) = \frac{R \times D \times C}{2 \times \overline{R + S} \times D - C \times R}$, & exinde inveniri poſſint puncta ad involutas pertinentia, &c.

Sit curva epicyclois, & conſequenter $D = C$; unde $PE = \frac{R \times D}{R + 2S}$, & exinde QP & PE erunt in datâ ratione inter ſe, & involuta etiam erit epicyclois.

Cor. 3. Invenire aream epicurvoidis.

Per problema inveniri poteſt abſciſſa, & ejus correſpondens ordinata, & conſequenter productum ex ordinatâ in fluxionem abſciſſæ ductâ, cujus fluens erit area ipſa.

Fig. 38. Cor. 4. Invenire aream inter curvam, quæ ſit baſis, & epicurvoidem contentam.

Ducantur duo circuli (LM & GH) ejuſdem curvaturæ ac curvæ revolventis, & baſis, ſuper quam progreditur curva revolvens; cum vero curva revolvens tangat baſim in puncto P, ſit a punctum epicurvoidem deſcribens, rotetur curva a puncto P ad punctum p, & per p ducatur linea pa' parallela lineæ Pa; & linea pb æqualis lineæ Pa, & eundem angulum cum baſis radio pC faciens, quem linea Pa, facit cum radio PC baſeos ad punctum contactûs P; cum vero curva revolvens ponatur ad punctum p, ſit l punctum curvam deſcribens; incrementum areæ epicurvoidis inter lineas Pa, Pp, pl, la contentæ, optime diſtingui poteſt in tres partes; quarum una eſt parallelogrammum inter lineas aP & pa' contentum, ſecunda vero triangulum inter lineas pa', pb & ab contentum, & tertia triangulum inter lineas pb, pl & bl contentum.

Parallelogrammum ($Papa'$) æquale eſt baſi $Pp = \dot{A}$ ductæ in perpendiculum $(an = P) = P\dot{A}$.

Triangulum $a'pb$ æquale eſt fractioni $\frac{\overline{aP}^2}{2CP} \times Pp(\dot{A})$; in quâ pro aP ſcribe D, & pro CP, R; & fit $\frac{D^2}{2R} \times \dot{A}$.

Ducatur

Ducatur perpendiculum ($d'r$) a puncto d' in lineam ph; duo triangula $d'pr$ & CpP ſunt ſimilia, angulus enim PCp æqualis eſt angulo $d'pr$, & conſequenter perpendiculum $d'r = \frac{d'p}{CP} \times Pp$, & area trianguli $d'ph = \frac{1}{2} d'p \times d'r = \frac{\overline{aP}^2}{2CP} \times Pp$.

Triangulum phl æquale eſt rectangulo $\frac{1}{2} ph \times hm$ perpendiculum a puncto h in lineam pl; ducatur linea lq tangens internum circulum lS in l, in tangentem lq ducatur perpendiculum pq a puncto p; erit $hm = \frac{pq \times hl}{pl}$ & $hl = \frac{Pp \times El}{PE}$, ubi E ſit centrum circulorum LM & lS; unde $\frac{1}{2} ph \times hm = \frac{pq \times El}{2PE} \times Pp$, & exinde ſumma prædictarum trium partium erit $\overline{P + \frac{D^2}{2R} + \frac{pq \times El}{2PE}} \times \dot{A}$, cujus fluens eſt area quæſita.

Fig. 76. & 38. Ex. Sint duæ curvæ GH & LM circuli, quorum radii CP & PE ſint R & r, & ſint etiam l punctum epicycloidem deſcribens, linea ql tangens circulum (LM) ad punctum l, Pn tangens circulos ad punctum contactûs P, qP perpendiculum ad tangentem lq & PI diameter circuli LM; ducantur lineæ El & ln perpendiculum in tangentem Pn; ob ſimilia triangula lPI & lPn, erit $PI\,(2r) : lP\,(D) :: lP : ln\,(P)$; unde $P = \frac{D^2}{2r}$; ſed $Pq = ln = \frac{D^2}{2r}$, & $El = PE$ quoniam radii ſunt ejuſdem circuli; unde $\frac{pq \times El}{2PE} = \frac{D^2}{4r}$, & $P + \frac{D^2}{2R} + \frac{pq \times El}{2PE} = \frac{D^2}{2r} + \frac{D^2}{2R} + \frac{D^2}{4r} = \frac{\overline{3R + 2r} \times D^2}{4Rr}$: ſit Pp incrementum arcûs circuli GH, & hl incrementum arcûs circuli LM, erit per hypotheſim $Pp = hl$: ducatur linea lP, & a puncto h agatur linea mh perpendiculum in lineam lP; ob ſimilia triangula PIl & lhm erit $PI\,(2r) : Pl\,(D) :: lh\,(Pp) : mh$; unde $mh = \frac{D \times Pp}{2r}$, &

area trianguli $lPh = \frac{(D)\, lP}{2} \times hm = \frac{D^2 \times Pp}{4r}$; ſed per coroll. $\overline{P + \frac{D^2}{2R} + \frac{pq \times El}{2PE}} \times Pp = \frac{\overline{3R + 2r} \times D^2}{4Rr} \times Pp$ incrementum areæ epicycloidis inter epicycloidem & circulum, qui ſit baſis, contentæ; unde incrementum areæ ſectoris circuli $\left(lPm = \frac{D^2 \times Pp}{4r}\right)$ erit ad cotemporaneum incrementum epicycloidis prædictum $\left(\frac{\overline{3R + 2r} \times D^2}{4Rr} \times Pp\right) :: R : 3R + 2r$, & conſequenter in datâ ratione.

Cor. 5. Utrum ſint quædam epicurvoidum areæ, &c. quæ exprimi poſſunt in algebraicis terminis datarum linearum, necne; e principiis in cor. 5. prob. 16. traditis inveniri poteſt.

Fig. 35. cor. 6. Invenire, utrum epicurvois ſit algebraica curva, necne.

E prædictis conſtat (z) epicurvoidis abſciſſam eſſe $LV - SA = \sqrt{LP^2 - \frac{\overline{PQ \times SR - LQ \times LS}^2}{LR^2}} - SA$, & ejus ordinatam (y) eſſe $LS + \frac{PQ \times SR - LQ \times LS}{LR}$; harum duarum quantitatum $(z \,\&\, y)$ termini vel ſunt algebraici vel fluxionales, &c.; ſi ſint algebraici, tum datur inter eas $(z \,\&\, y)$ algebraica relatio, & curva eſt algebraica curva; ſi vero ſit fluxionalis, tum poſtea in libro de fluxionibus dabitur ſolutio; &c.

Hinc conſtat, ſi modo curvæ rotantes & earum baſes ſint algebraicæ & rectificari poſſint, earum epicurvoides eſſe curvas algebraicas. Et ſic de reliquis proprietatibus.

LEMMA.

Fig. x. 1. Sint AB & Ab arcus quam minimi curvarum, eodem tempore deſcripti ~~ex~~ a corporibus quibuſcunque motibus vel viribus agitatis; ſint AD & Ad rectæ tangentes prædictos arcus AB & Ab ad punctum A, & ultimo ad arcus ipſos rationem æqualitatis habentes; ducantur lineæ

DH

DH & dH parallelæ lineis Ad & AD & ſeſe interſecantes in puncto H; etiamque linea AH; erit AH linea tangens curvam hiſce motibus vel viribus conjunctim deſcriptam.

Sint S & R reſpective curvarum AB & Ab ad punctum A radii curvaturæ; a punctis D & d ducantur lineæ DBE & bde perpendiculares ad tangentem AH; tum erit ſagitta (Bp) arcûs AB perpendicularis ad tangentem AD, ad ſagittam (BD) ejuſdem arcûs cujus directio perpendicularis eſt ad lineam $AH :: AE : AD$; at ſagittæ perpendiculares ad tangentem erunt ultimo in ratione æqualitatis cum $\frac{AD^2}{2S} = \frac{AB^2}{2S}$; unde ſagittæ DB, quæ productæ ſunt perpendiculares ad lineam AH, erunt ultimo $= \frac{AD^3}{2S \times AE}$; & ſic probari poteſt ſagittam bd eſſe ultimo in ratione æqualitatis cum $\frac{Ad^3}{2R \times Ae}$: ſit HL ſagitta, ad lineam AH perpendicularis, curvæ AL a conjunctis prædictis motibus vel viribus deſcriptæ; tum erit ultimo $HL = DB \pm db$; & exinde radius curvaturæ curvæ AL ad punctum $(A) = \frac{AH^2}{2HL} = \frac{AH^2}{2\overline{DB \pm db}}$

$$= \frac{AH^2}{\frac{AD^3}{S \times AE} \pm \frac{Ad^3}{R \times Ae}} = \frac{AH^2 \times S \times R \times AE \times Ae}{AD^3 \times Ae \times R \pm Ad^3 \times AE \times S}.$$

Ex eâdem prorſus methodo inveniri poteſt radius curvaturæ ad quodlibet punctum curvæ a tribus vel pluribus viribus & motibus conjunctis deſcriptæ; & ex iiſdem principiis inveniri poſſunt ejus curvaturæ diverſorum generum variationes.

Fig. z. 2. Duæ curvæ LM & RS in punctis earum contactuum ſeſe ſemper tangant; ſit P punctum curvæ RS, quod tangit curvam LM in puncto Q; ita progrediatur curva RS ſuper curvam LM revolvendo, ut arcus QP curvæ LM ſit ad arcum QP curvæ RS, revolvendo, dum tranſeat curva RS a puncto P curvæ LM ad punctum Q ejuſdem curvæ, deſcriptum $:: m : n$ in datâ ratione.

Sint

Sint $OP\ (R)$, $oP\ (S)$ radii, & O & o centra circulorum earundem curvaturarum, quas habent datæ curvæ LM & RS in punčto P; in lineâ oPO aſſumatur punčtum p, per id ducantur circuli ps & pm, quorum radii ſunt op & Op & centra o & O reſpective: ſi circuli, quorum radii ſint oP & op, ſimul ferantur circa punčtum vel centrum o; erit velocitas (V) punčti P ad velocitatem punčti p :: $oP\ (S) : oP + Pp\ (z)$, unde velocitas punčti p circa centrum o erit $= \frac{V \times \overline{S+z}}{S}$: &, ſi modo circuli, quorum radii ſint PO & pO, ſimiliter ferantur circa centrum O; erit velocitas punčti p ad velocitatem punčti P :: $Op : OP\ (R)$; at per hypotheſim velocitas punčti P circa centrum (O) eſt ad ejus velocitatem circa centrum (o) :: $m : n$, unde velocitas punčti P circa centrum $O = \frac{m \times V}{n}$, & exinde velocitas punčti p circa idem centrum $(O) = \frac{m \times V \times Op}{n \times OP} = \frac{m \times V \times \overline{OP - Pp}}{n \times OP} = \frac{m \times V \times \overline{R-z}}{nR}$: fingatur velocitas punčti p circa centrum O eadem eſſe ac ejus velocitas circa centrum o; i. e. $\frac{V \times \overline{S+z}}{S} = \frac{mV \times \overline{R-z}}{nR}$; unde $Pp = z = \frac{\overline{m-n} \times RS}{nR + mS}$.

Sit l datum punčtum cum curvâ RS revolvens; tum erit linea pl perpendicularis ad curvam generatam a punčto l.

Radius curvaturæ curvæ generatæ a punčto l erit $\frac{2 \times \overline{R+S} \times D^2}{2 \times \overline{R+S} \times D - C \times \overline{R-z}}$, ubi $D = pl$, & $C =$ chordæ curvaturæ circuli qs in punčto p & directione pl.

Hoc ſequitur e ſcribendo $R - z$ & $S + z$ pro R & S in expreſſione pro radio curvaturæ prius traditâ $\frac{2 . \overline{R+S} \times D^2}{2 \times \overline{R+S} \times D - C \times R}$.

PROB.

PROB. XVIII.

Fig. 39. Iiſdem poſitis, & datis quibuſlibet curvis (*LM*, *HK*, *QO*, &c.) ſuper ſeſe revolventibus, rectificare curvam a puncto in extremâ curvâ generatam.

Sint velocitates curvarum ſuper ſeſe revolventium reſpective *m*, *n*, *r*, *s*, *t*, &c.

Sit prima curva *LM* baſis, ſuper quam progreditur curva *HK*; rectificentur hæ curvæ inter ſe, & ducantur circuli, quorum radii ſunt *CP* & *PD* reſpective, eandem curvaturam habentes ac datæ curvæ in puncto contactûs *P*; ſit *Q* punctum deſcribens epicurvoidem, ducatur linea *DQ* jungens centrum curvaturæ ſecundæ curvæ *HK* & punctum *Q*, cum radio *DQ* deſcribatur circulus (*NQ*, &c.) ducatur etiam linea *PQ*; lineæ *DQ* agatur *Pr* parallela; & ſint *Qo*, *Qm* & *Qn* cotemporanea incrementa lineæ *CQ*, arcûs circuli *QN*, & epicurvoidis generatæ a puncto *Q*; tum propter ſimilia triangula *Qmn* & *PrQ* erit $Qn : Qm :: QP : Pr$; &, quoniam incrementa (*Qm* & *Pp*) cotemporaneorum arcuum circulorum (*QN* &c. & *HK*) ſunt inter ſe in ratione ſuorum radiorum *DN* & *PD*, erit $Qm : Pp :: DN : DP$; quibus rationibus compoſitis, reſultat proportio $Qn : Pp :: QP \times DN : Pr \times DP$, i. e. $Qn : Pp :: QP \times CD : CP \times DP$, i. e. velocitas puncti *Q* deſcribentis epicurvoidem eſt ad velocitatem ſecundæ curvæ progredientis ſuper primam in ratione $QP \times CD : CP \times DP$; deinde, ut prius, progrediatur curva *HK* cum puncto *k* ſuper baſim *LM*, & ſit *k* punctum primam epicurvoidem deſcribens, agatur linea *kP*; tum erit velocitas (*Pp*) ſecundæ curvæ *HK* ſuper primam *LM* progredientis ad velocitatem puncti *k* epicurvoidem deſcribentis $:: CP \times DP : CD \times kP$: nunc fingatur ſecunda curva *HPKwZz* quieſcere, & ſit *Z* punctum contactûs ſecundæ & tertiæ curvarum, progrediatur tertia curva *QO* ſuper ſecundam *wZz*, & ducatur circulus, cujus radius ſit *ZT*, eandem habens curvaturam ac data curva *QO* in puncto contactûs *Z*; ſit *k* punctum epicurvoidem ſecundam deſcribens;

bens; tum erit velocitas puncti k ſecundam epicurvoidem deſcribentis ad velocitatem tertiæ curvæ ſuper ſecundam progredientis :: $ST \times Zk : ZS \times ZT$; ſi modo ZS ſit radius curvaturæ ſecundæ curvæ $HPKwZz$ ad punctum contactûs ſecundæ & tertiæ curvarum: ſed velocitas ſecundæ curvæ ſuper primam progredientis per hypotheſim eſt ad velocitatem tertiæ curvæ ſuper ſecundam progredientis :: $m : n$; ergo erit velocitas puncti k deſcribentis ſecundam epicurvoidem ad velocitatem ejuſdem puncti k primam epicurvoidem deſcribentis :: $n \times ST \times Zk \times CP \times DP : m \times ZS \times ZT \times CD \times kP :: N : M$: erunt Pk & Zk reſpective perpendiculares ad primam & ſecundam epicurvoidem: in lineis Pk & Zk aſſumantur lineæ kh & kl quantitatibus M & N proportionales, & ducantur lineæ ht & lt reſpective parallelæ lineis kl & kh; erit diagonalis kt perpendicularis ad epicurvoidem a puncto k ex his conjunctis motibus, viz. motu ſecundæ curvæ HK ſuper primam LM, & tertiæ QO ſuper ſecundam HK, deſcriptam; unde facile conſtat epicurvoidis incrementum.

Eadem principia applicari poſſint ad plures curvas ſuper ſeſe revolventes.

Sint H & K reſpective radii curvaturæ primæ & ſecundæ epicurvoidis, qui e formulâ prius datâ $\frac{2 \times \overline{R+S} \times D}{2 \cdot \overline{R+S} \cdot D - CR}$ facile conſtabunt; tum, ſi modo hp & lq ducantur perpendiculares ad lineam kt; erit radius curvaturæ curvæ ~~ex~~ a corpore his conjunctis motibus agitato generatæ per lemma $= \frac{kt^2 \times H \times K \times kp \times kq}{kh^3 \times kq \times K \pm kl^3 \times kp \times H}$.

Eadem principia facile applicari poſſint ad inveniendos curvaturæ radios epicurvoidum a pluribus curvis ſuper ſeſe revolventibus generatarum.

Hoc problema aliter e principiis prius traditis reſolvi poteſt.

Per methodos in præcedenti problemate expoſitas inveniri poſſunt area, involutum, &c. epicurvoidis prædictæ.

PROB.

PROB. XIX.

Datis omnibus præter unam curvis ſuper ſeſe revolventibus, earum velocitatibus, & curvâ generatâ ex earum rotatione; invenire curvam incognitam.

His datis; datur diſtantia inter duas curvas, inter quas interponitur quæſita curva, i. e. datur chorda vel ejus incrementum quæſitæ curvæ & ejus arcûs incrementum; unde inveniri poteſt curva quæſita.

Fig. 40. Ex. Datâ curvoide, datur relatio inter abſciſſam & ejus correſpondentem ordinatam, & conſequenter inter Qn perpendiculum in curvam, QF ejus ſubnormalem & nF: ſit (MPL) curva quæſita; tum MP chorda quæſitæ curvæ, MI perpendiculum in tangentem & PI erunt reſpective æquales lineis Qn, nF & QF; conſequenter datur relatio inter chordam, perpendiculum in tangentem, & tangentem quæſitæ curvæ; unde inveniri poteſt curva quæſita.

Et ſic reſolvi poſſunt plura hujuſce generis problemata.

PROB.

Fig. y. Invenire curvam deſcriptam a puncto vel intra vel extra datam curvam poſito, cujus peripheria per datum punctum D tranſeat, & in eo puncto quamcunque rectam tangat.

Ex. 1. Sit curva circulus AB, cujus centrum ſit C; ejus peripheria per punctum D ſemper tranſeat, & rectam EF tangat: ſit e punctum deſcribens curvam quæſitam.

Facile conſtat curvam quæſitam eſſe circulum, cujus centrum eſt C & radius Ce.

Ex. 2. Sit data curva ellipſis, cujus peripheria per punctum D ſemper tranſeat, & rectam EF in puncto D tangat, & ſit centrum ellipſeos punctum curvam deſcribens; ſcribantur pro ellipſeos majore & minore axibus reſpective $2t$ & $2c$, & pro abſciſsâ curvæ quæſitæ a puncto D in lineâ EF incipiente, & ejus ordinatâ ad rectos angulos ad abſciſſam poſitâ, reſpective z & v; tum erit æquatio quæſita $\overline{t^2+c^2}\, v^2 - c^2t^2 = v^4 + v^2z^2$.

Sit enim Py recta tangens ellipſim in puncto P, & Cy perpendiculum a centro C in tangentem Py; tum, ſi Py ſit abſciſſa z, erit

Cy

Cy ejus correſpondens ordinata; unde haud difficile erit æquationem prius traditam inveſtigare.

Cor. Ex hâc æquatione facile deduci poteſt, ſi $Py = z$ ſit maximum, fore $Py = t - c$ & $Cy = \sqrt{t \times c}$.

Ex. 3. Iiſdem poſitis; ſit focus ellipſeos punctum deſcribens curvam, tum erit æquatio ad curvam quæſitam $v^6 + \overline{2c^2 - 4t^2 + z^2}\, v^4 + \overline{c^4 + 2c^2 z^2}\, v^2 + c^4 z^2 = 0$..

Si in his æquationibus pro c^2 ſcribatur $-c^2$; evadet æquatio ad curvam, quam deſcribit centrum vel focus hyperbolæ.

Ex. 4. Sit curva revolvens parabola, cujus æquatio inter abſciſſam (x) ad axem & ejus ordinatas (y) relationem deſcribens, ſit $px = y^2$; tum, ſi z & v reſpective denotent abſciſſam prædictam & ejus ordinatas curvæ quæſitæ, & focus parabolæ ſit punctum curvam deſcribens, erit $16\, v^4 - p^2 v^2 = p^2 z^2$.

In genere inveniatur æquatio relationem exprimens inter abſciſſam x in lineâ PT contentam & ejus ordinatam $y = PM$ ad rectum angulum in abſciſſam inſiſtentes, & ſit a punctum curvam deſcribens in abſciſsâ poſitum; tum erit $\frac{y\dot{x}}{\dot{y}} = PT$ ſubtangenti, & $\frac{y^2 \dot{x}^2}{\dot{y}^2} + y^2 = TM^2$; ſed ex ſimilibus triangulis (ſi modo ay ſit perpendicularis ad tangentem TMy) erit $TM : PM\,(y) :: aT = PT \mp aP\,(x \mp A) : ay = v$ ordinatam curvæ quæſitæ; et ſimiliter $TM : TP :: aT : TM \mp My\,(z)$, ubi $My = z$ eſt abſciſſa curvæ quæſitæ; æqualibus inter ſe ſuppoſitis rectangulis ſub extremis & mediis in ſeſe ductis, reducantur duæ æquationes exinde reſultantes & æquatio ad datam curvam in unam, ita ut exterminentur duæ incognitæ quantitates x & y & earum fluxiones; reſultat æquatio relationem inter z & v abſciſſam & ejus ornatas curvæ quæſitæ deſignans.

PROB. XX.

Fig. 41. Datâ curvâ (LMN); invenire alteram $(LRST)$, cujus (ſi abſciſſa LP eadem ſit ac datæ curvæ abſciſſa) ordinata erit proportionalis areæ $(LMCL)$ datæ curvæ circa datum punctum C deſcriptæ.

Scribatur

Scribatur pro abſciſſa (LP) x, pro ordinatâ PM (y), & pro (LC) a; area circa datum punctum C deſcripta = areæ $LPML$ — areâ trianguli MCP; ergo fluxio areæ circa punctum C = flux. areæ $LPML$ $(y\dot{x})$ — flux. triang. $MCP\left(\frac{x\dot{y}+y\dot{x}-a\dot{y}}{2}\right)=\frac{y\dot{x}-x\dot{y}+a\dot{y}}{2}$; cui fluxioni proportionalis eſſe debet fluxio curvæ quæſitæ ordinatæ ($\dot{v}$), ſi modo ejus abſciſſa ſit (x); inveniatur igitur fluens hujus fluxionis, erit proportionalis ordinatæ curvæ quæſitæ.

Cor. 1. Sit data curva circulus, cujus æquatio ſit $y^2=2rx-x^2$, erit $y\dot{x}-x\dot{y}+a\dot{y}=\sqrt{2rx-x^2}\,\dot{x}-\frac{\overline{rx-x^2}\times\dot{x}}{\sqrt{2rx-x^2}}+a\dot{y}=\frac{r\dot{x}x}{\sqrt{2rx-x^2}}+a\dot{y}=2\dot{v}$, quæ eſt æquatio ad trochoidem; conſequenter per trochoidem ſecari poteſt area circuli vel ellipſeos proportionalis quibuſcunque datis quantitatibus, quam propoſitionem primum invenit Walliſius.

SCHOLIUM.

Variæ dantur methodi deſcribendi curvas vel per motum progreſſivum vel circumlatitium: motus autem progreſſivi cum progreſſivis, progreſſivi cum circumlatitiis & circumlatitii cum circumlatitiis componi poſſint & decomponi modis innumeris: & ſic e quamplurimis curvarum proprietatibus conſequatur earum deſcriptio.

Hinc fere innumeræ conicarum ſectionum deſcriptiones e pluribus earum proprietatibus erui poſſint: de paucis ſuperiorum ordinum curvis dantur conſtructiones, utpote paucæ quidem inveniuntur earum proprietates, quæ haud ad duo vel plura curvæ puncta ſimul adjuncta reſpiciunt.

Recentiores diſtinxere curvas in algebraicas & fluxionales, priores vero in varios ordines ſecundum dimenſiones earum æquationum; ita probe diſtingui poſſint fluxionales curvæ in ordines ſecundum ordines fluxionum, quas neceſſario involvant: natura autem infinite plures admittat, quæ neque in hunc neque in illum claſſem reduci poſſint, ut conſtat ex ultimo capite hujuſce libri, &c.

Fluxionales curvæ conſtrui poſſint vel ex aliis fluxionalibus curvis, vel e rotatione curvarum ſuper alias; conſtrui autem poſſint ex aliis fluxi-

fluxionalibus curvis ejuſdem ordinis & algebraicis curvis eodem modo, quo una algebraica curva conſtruitur ex aliis & rectis lineis: deſcribi vero poſſint fluxionales curvæ primi ordinis e rotatione algebraicarum curvarum ſuper alias, & ex algebraicis curvis; deſcribi vero poſſint fluxionales curvæ ſecundi ordinis e rotatione fluxionalium curvarum primi ordinis ſuper alias, vel e rotatione algebraicarum ſuper fluxionales curvas primi ordinis, vel e rotatione fluxionalium curvarum primi ordinis ſuper algebraicas, & fluxionalibus primi ordinis & algebraicis curvis; & ſic deinceps; quod facile conſtabit cuicunque has res ſerio perpendenti; deſcribi etiam poſſint fluxionales curvæ e continuis calculis per approximationem, ſed hæc ad infinitas ſeries referri debent.

CAP. III.

Algebraicorum Solidorum Proprietates.

Fig. 45. DATA æquatione relationem exprimente inter abſciſſam ($AP = x$) datæ curvæ (MMM &c.) & ejus correſpondentes ordinatas ($MP = y$) ad rectum angulum (APM) ſuam abſciſſam ſecantes; invenire æquationem relationem definientem inter abſciſſam ($Lp = z$), ubi L eſt punctum in abſciſsâ AP poſitum, & ejus correſpondentes ordinatas ($pm = v$) abſciſſam Lp ad rectum angulum Lpm ſecantes, cujuſcunque ſectionis a dato plano factæ ſolidi e converſione datæ curvæ circa axem ſuum generati.

Sit $r : s :: Lp\ (z) : LP$, & $r^2 : t^2\ (r^2 - s^2) :: z^2 : Pp^2$; ſed differentia $PM^2\ (y^2) - Pp^2 \left(\frac{t^2 z^2}{r^2}\right)$ æqualis eſt quadrato lineæ vel ordinatæ (v) per punctum p ductæ; unde erit datæ curvæ abſciſſa $AP = x = a\ (AL) + \frac{sz}{r}$, & ejus ordinatæ quadratum $PM^2 = y^2 = v^2 + \frac{t^2 z^2}{r^2}$: ſubſtituantur hæ quantitates $a + \frac{sz}{r}$ & $v^2 + \frac{t^2 z^2}{r^2}$ pro ſuis valoribus (x & y^2) in datâ æquatione, reſultat æquatio relationem inter abſciſſam ($Lp = z$) requiſitæ ſectionis & ejus ordinatas (v) exprimens.

Si vero angulus haud ſit rectus; tum e prob. primo ita transformari poteſt data æquatio, ut exprimat relationem inter abſciſſam & ejus ordinatas rectum angulum cum eâ continentes.

Cor.

Cor. 1. Sectio a quocunque plano facta ſolidi a rotatione algebraicæ curvæ n ordinis circa axem ſuum generati, cujus æquatio ſit $(Ay^{n+1} + \overline{a+bx}\,y^{n} + \&c. = 0,)$ haud poteſt eſſe curva majoris quam $2n+1$ ordinis. Sivero ſint nullæ impares ordinatæ (y) dimenſiones in datâ algebraicæ curvæ æquatione; tum prædicta ſectio non poteſt eſſe curva majoris quam n ordinis. Subſtituantur enim in datâ æquatione $n+1$ dimenſionum, i. e. æquatione curvæ n ordinis pro x ejus valor $a + \frac{sz}{r}$, & pro y ejus valor $\sqrt{v^2 + \frac{t^2 z^2}{r^2}}$ in priori caſu, & æquationis reſultantis exterminetur ſurda quantitas $\sqrt{v^2 + \frac{t^2 z^2}{r^2}}$; haud majores invenientur dimenſiones quantitatum x & y in reſultanti æquatione quam $2n+2$, i. e. curva haud altioris ordinis erit quam $2n+1$. In ſecundo vero caſu in datâ æquatione pro x ſcribatur $a + \frac{sz}{r}$, & pro y^2 ſubſtituatur $v^2 + \frac{t^2 z^2}{r^2}$; reſultans æquatio haud majores habet dimenſiones quam data.

Cor. 2. Sectiones ſolidi generati e converſione conicæ ſectionis circa ejus principalem axem erunt conicæ ſectiones; ellipſeos ellipſis; parabolæ, ellipſis vel parabola, prout ſectio axem ſuum ſecat, vel ei eſt parallela; hyperbolæ ellipſis, parabola vel hyperbola, prout ſecat duas aſymptotos ex eâdem parte, parallela eſt uni aſymptoto vel ſecat oppoſitas hyperbolas.

Cor. 3. Sectiones ſolidi generati a rotatione conicæ hyperbolæ circa axem ſuum erunt ſimiles ſectionibus coni, cujus latera ſunt hyperbolæ aſymptoti, & cujus ſectiones eundem faciunt angulum cum axe coni, quem habent hyperbolæ ſectiones; ſolidi vero ſectiones a rotatione parabolæ circa axem ſuum generati, erunt ſimiles ſectionibus recti cylindri, quæ eundum angulum cum lateribus vel axe cylindri, ac ſectiones conicæ parabolæ cum ſuo axe faciunt.

Fig. 43. Generaliter vero ſint duæ conicæ ſectiones $(ABCD$ & $EFGH)$, quarum duæ æquationes relationes exprimentes inter abſciſſas $(z$ & $w)$ curvarum ad axes & earum correſpondentes ordi-

 natas

natas (v & u) ſunt reſpective $v^2 = \frac{c^2}{t^2} \times \overline{t^2 \pm z^2}$ & $u^2 = \frac{C^2}{T^2} \times \overline{T^2 \pm w^2}$; rotentur hæ curvæ circa ſuos axes, & ſolidi generati a rotatione curvæ ($ABCD$) ſit ſectio ($\alpha\,\beta\,\gamma\,\delta$) ſimilis ſectioni ($\epsilon\,\zeta\,\eta\,\theta$) ſolidi a rotatione curvæ ($EFGH$) generati.

Prior vero ſectio cum axe ſui ſolidi faciat angulum, cujus ſinus eſt S; poſterior vero cum axe ſui ſolidi angulum, cujus ſinus eſt s; tum erit $S^2 = \frac{\overline{1 \pm \frac{C^2}{T^2} \times s^2 \pm \frac{c^2}{t^2} \mp \frac{C^2}{T^2}}}{1 \pm \frac{c^2}{t^2}}$: ſubſtituantur enim pro (z & v^2) per problema in priori æquatione $\sqrt{1 - S^2} \times x$ (ſectionis abſciſſam) $+ a$, & y^2 (ejus ordinatæ quadratum) $+ S^2 x^2$; in poſteriori æquatione pro (w & u^2) ſcribantur reſpective $\sqrt{1 - s^2} \times \zeta$ (ſectionis abſciſſam) $+ e$, & ξ^2 (ejus ordinatæ quadratum) $+ s^2 \zeta^2$; reſultant duæ æquationes $y^2 + S^2 x^2 = c^2 \pm \frac{c^2 \times \overline{1 - S^2}}{t^2} x^2 +$ &c. $\xi^2 + s^2 \zeta^2 = C^2 \pm \frac{C^2 \times \overline{1 - s^2}}{T^2} \zeta^2 +$ &c. ſed quoniam hæ duæ ſectiones inter ſe ſunt ſimiles, coefficiens quadrati ordinatæ ad coefficientem quadrati abſciſſæ ſuæ in utrâque æquatione eandem habet rationem, & conſequenter $S^2 \mp \frac{c^2 \times \overline{1 - S^2}}{t^2} = s^2 \mp \frac{C^2 \times \overline{1 - s^2}}{T^2}$; reducatur hæc æquatio, invenietur $S^2 = \frac{\overline{1 \pm \frac{C^2}{T^2} \times s^2 \mp \frac{C^2}{T^2} \pm \frac{c^2}{t^2}}}{1 \pm \frac{c^2}{t^2}}$.

Fig. 44. Cor. 4. Conica ſectio BCN circa ſuum axem (BC) rotetur, & ſit ſectio ſolidi generati DEF; ducatur plano (DEF) parallelum planum HIK, tangens ſolidum in puncto I: biſecetur axis ſectionis DF in L; linea IL producta tranſit per centra omnium ſectionum ſectioni (DEF) parallelarum.

Similiter ſit Q focus conicæ ſectionis, cujus rotatione circa axem ſuum BC generatur ſolidum; ſit DEF ſectio hujus ſolidi, cui ſit planum HIK tangens ſolidum in puncto I parallelum; ſit linea IN diameter a puncto contactûs I per centrum ſolidi tranſeuns; ſuper lineam

neam IN tanquam diametrum per punctum $\mathcal{Q}$ deſcribatur conica ſectio, quæ tangat planum HIK; hæc conica ſectio per focos omnium ſectionum ſectioni DEF parallelarum tranſibit.

Hinc focus omnis ſectionis prædicti ſolidi per punctum $\mathcal{Q}$ focum conicæ ſectionis, cujus rotatione generatur ſolidum, tranſeuntis, erit etiam punctum $\mathcal{Q}$ focus conicæ ſectionis prædictæ; & omnium prædictarum ſectionum latera recta principalia erunt æqualia.

Demonſtratio. Sint $2t$ & $2c$ major & minor axis ellipſeos, cujus rotatione generatur ſolidum, & $2b\,(MP)$ major axis ſectionis MOP per centrum O ſphæroidis tranſeuntis & plano HIK parallelæ, ejus vero minor axis erit $2c$: pro IO ſcribatur a; tum per bene cognitam ellipſeos proprietatem erit $t^2 + c^2 = a^2 + b^2 = A$, etiamque (ſi Ir ſit linea perpendicularis ad BC) $Or^2 = x^2 = \frac{a^2 - c^2}{t^2 - c^2} \times t^2$; $OH^2 = \frac{t^4}{x^2} = \frac{t^2 - c^2}{a^2 - c^2} \times t^2$; $Hr = OH - x = \frac{t \times \overline{t^2 - a^2}}{\sqrt{t^2 - c^2} \times \sqrt{a^2 - c^2}}$; ſed erit $t^2 : c^2 :: \overline{t^2 - x^2}\left(t^2 \times \frac{\overline{t^2 - a^2}}{t^2 - c^2}\right) : Ir^2 = c^2 \times \frac{t^2 - a^2}{t^2 - c^2}$; unde $Hr^2 + Ir^2 = HI^2 = \frac{t^2 - a^2}{t^2 - c^2} \times \frac{t^4 - t^2a^2 + a^2c^2 - c^4}{a^2 - c^2} = \overline{t^2 - a^2} \times \frac{t^2 + c^2 - a^2}{a^2 - c^2}$; ex ſimilibus triangulis OIH & $OT\mathcal{Q}$ ſequitur proportio $OH^2\left(\frac{t^2 - c^2}{a^2 - c^2} \times t^2\right) : HI^2 \left(\overline{t^2 - a^2} \times \frac{t^2 + c^2 - a^2}{a^2 - c^2}\right) :: O\mathcal{Q}^2\,(t^2 - c^2) : \mathcal{Q}T^2 = \frac{\overline{t^2 - a^2}}{t^2} \times \overline{t^2 + c^2 - a^2}$, etiamque $OH^2\left(\frac{t^2 - c^2}{a^2 - c^2} \times t^2\right) : IO^2\,(a^2) :: \overline{t^2 - c^2}\,(O\mathcal{Q}^2) : OT^2 = \frac{a^2}{t^2} \times \overline{a^2 - c^2}$; & exinde $OI^2 - OT^2 = a^2 - \frac{a^2}{t^2} \times \overline{a^2 - c^2} = \frac{a^2t^2 - a^4 + a^2c^2}{t^2}$: ſed $OI^2\,(a^2) : OM^2\,(t^2 + c^2 - a^2) :: IO^2 - OT^2 = \frac{a^2t^2 - a^4 + a^2c^2}{t^2} : TR^2 = \frac{\overline{t^2 + c^2 - a^2}^2}{t^2}$: omnes ſphæroidis ſectiones (MP & RS) inter ſe parallelæ, erunt inter ſe ſimiles; conſequenter in utriſque (MP & RS) eadem eſt ratio, quam habet major axis ad diſtantiam centri a foco; ſed in ſectione MP per centrum tranſeunte hæc ratio

eſt $OM\,(\sqrt{t^2+c^2-a^2}) : \sqrt{MO^2-c^2}\,(\sqrt{t^2-a^2})$; ergo erit $\sqrt{t^2+c^2-a^2} : \sqrt{t^2-a^2} :: (TR)\,\frac{t^2+c^2-a^2}{t} : \frac{\sqrt{t^2-a^2}}{t} \times \sqrt{t^2+c^2-a^2}$ diſtantiam centri T a foco ſectionis RS tranſeuntis per focum ellipſeos ſolidum generantis: ſed hæc quantitas prius probatur diſtantiæ QT æqualis; ergo Q etiam erit focus prædictæ ſectionis RS. *Q. E. D.*

Et ſic inveniri poſſunt centra, foci, &c. ſectionum a planis circa data puncta rotantibus factarum, & conſequenter loci centrorum, focorum, &c.

Cor. 5. Si per ovalem generalem deſignetur quæcunque curva in ſeſe redeuns, & haud in infinitum progrediens; per parabolam vero $\frac{n}{m}$ generum, curva, cujus æquatio ad infinitam diſtantiam deſignatur per æquationem $y = ax^{\frac{n}{m}}$; ubi x denotat abſciſſam, y vero ordinatam; per hyperbolam $\frac{n}{m}$ generum, curva, cujus æquatio ad infinitam diſtantiam deſignatur per æquationem $y = \frac{a}{x^{\frac{n}{m}}}$: tum ſectio a quocunque plano facta omnis ſolidi e converſione ovalis circa ſuum axem generati erit ovalis: parabolæ $\frac{n}{m}$ generum erit ovalis vel parabola conici vel $\frac{n}{m}$ generum; ſi vero linea circa quam gyretur prædicta parabola ſit ejus axis, tum prædicta ſectio vel erit ovalis vel parabola $\frac{n}{m}$ generum: hyperbolæ $\frac{n}{m}$ generum e gyratione circa ſuam aſymptoton ſolidi generati ſectio ſolummodo poteſt eſſe ovalis vel hyperbola $\frac{n}{m}$ generum; ſi vero gyretur circa aliam lineam, tum poteſt eſſe ovalis, parabola conici generis vel hyperbola $\frac{n}{m}$ generum.

Si curva habet plura crura vel parabolica vel hyperbolica, eadem eſt methodus ratiocinandi de ſingulis.

In ſingulis hiſce æquationibus pro y ſcribatur $\sqrt{v^2 + \frac{t^2 z^2}{r^2}}$; & pro x, $a + \frac{sz}{r}$; ex æquationibus reſultantibus inveniri poſſunt crura quæſita.

Cor. 6. Nulla datur ſectio ſolidorum ſecundum prædictam methodum generatorum, quæ non habet diametrum, quæ ita dividat ejus ordinatas, ut earum affirmativi & negativi valores ſint inter ſe reſpective æquales.

Cor. 7. Ducantur quotcunque plana per axem AP tranſeuntia, & æquales angulos ſecum facientia; tum hæc plana dividunt ſolida generata in æquales partes.

Def. 1. Algebraica ſolida diſtingui poſſunt in ordines vel dimenſiones ſecundum maximas dimenſiones æquationum curvarum, quæ ſint ſectiones dati ſolidi.

Fig. 45. 2. In æquationibus ad ſolida numerus $(r + 2)$ incognitarum quantitatum binario ſuperat numerum (r) æquationum, e. g. æquatio ad ſolidum $APMm$, &c. neceſſario involvit tres incognitas independentes quantitates AP, Pp & mp; duæ vero æquationes involvunt quatuor incognitas quantitates; &c.

3. Sint AP prima abſciſſa, Pp ſecunda, & pm ordinata ad ſecundam abſciſſam, & pro lineis AP, Pp & pm ſcribantur reſpective x, z & y; æquatio ad ſolidum n dimenſionum erit hujuſce formulæ $Ay^n + \overline{a + bz + cx}\, y^{n-1} + \overline{d + ez + fx + gz^2 + hxz + kx^2}\, y^{n-2} + \&c. = 0$.

Cor. 1. E prob. noſt. Medit. Algebr. conſtat hanc æquationem $\overline{n+1} \times \frac{n+2}{2} \times \frac{n+3}{3} - 1$, incognitas quantitates habere.

Hinc ſolidum n dimenſionum duci poteſt per $\overline{n+1} \times \frac{n+2}{2} \times \frac{n+3}{3} - 1$ data puncta, vel $n + 1 \times \frac{n+2}{2} \times \frac{n+3}{3} - 1$ datas lineas tangens, &c.

Cor. 2. Sint quantitates $a = 0$, $b = 0$, & $c = 0$; planum tranſeuns per lineas AP & Pp dici poſſit diameter ſolidi; ſi enim (n) ſint diverſi ordinatæ $(pm = y)$ valores, planum prædictum ita dividet ſolida

ſolida generata, ut ſumma ſolidorum ex unâ parte plani æqualis ſit ſummæ ſolidorum ex alterâ.

Cor. 3. Si vero ſint algebraica ſolida eaſdem habentia dimenſiones ac data ſolida, & quæ dici poſſint horum ſolidorum aſymptoti eodem ſenſu, quo recta dicitur aſymptotus curvæ; tum omnis diameter ſolidorum aſymptotorum etiam erit diameter ipſorum ſolidorum, & differentia inter contenta aſymptotorum & ſolidorum utrinque inter ſe æqualis erit.

Cor. 4. Si duæ ſint æquationes n dimenſionum ad duo diverſa ſolida pertinentes; in his autem æquationibus ſint termini iidem, in quibus n & $n - 1$ dimenſiones inveniuntur; & ſint anguli, quos ſecunda abſciſſa cum primâ & ordinatæ faciunt cum ſecundâ abſciſsâ in utrâque æquatione, iidem; tum diameter unius ſolidi etiam erit diameter alterius; &c.

Cor. 5. Sint dimenſiones nullius termini vel nullius quantitatis x, y, z in datâ æquatione contentæ impares; ſi angulus, quem abſciſſæ ſecum & ordinatæ cum ſecundâ abſciſsâ faciunt, ſit rectus; erit vere diameter vel axis ſolidi.

Cor. 6. Axes ſolidorum inveniri poſſunt ex iiſdem principiis, ac axes curvarum, quæ docentur in theor. 6. transformetur enim æquatio relationem inter abſciſſas & earum ordinatas exprimens in æquationem relationem generaliter deſignantem inter quaſcunque alias abſciſſas (x, z) ſecum rectum angulum habentes, & earum ordinatas (y) ad rectum angulum in ſecundam abſciſſam (z) inſiſtentes: aſſumatur x tanquam invariabilis quantitas, & per theor. 6. inveniatur, utrum curva reſultans poſſit habere axes, quicunque ſit valor quantitatis x, necne: & conſit corollarium.

Hinc non poteſt datum ſolidum, cujus ſectiones ab eodem puncto ſuperficiei dati ſolidi incipiunt,habere majorem axium numerum,quam ſunt æquationis ad datum ſolidum dimenſiones; ni infinitos habeat axes; e methodo in curvis prius uſitatâ conſtat numerus axium hujuſce generis, quos recipiat quodcunque ſolidum, cujus datur æquatio; omnes proximi axes ſeſe in centro ſolidi ad æquales angulos interſecant.

Aſymptoti ſolidorum, parabolica ſolida, ovales eodem modo inveniri poſſint, quo in algebraicis curvis.

Fere

Fere omnia, quæ in primo capite de curvis deducuntur, æque mutatis mutandis de ſolidis hujuſce generis affirmari poſſunt.

PROB. XXII.

Datis duabus æquationibus ad duo ſolida, invenire curvam, quæ eſt communis ſectio ſuperficierum horum ſolidorum.

Ita transformentur duæ abſciſſæ (x & z) duorum ſolidorum, ut evadant eædem in utroque ſolido, & mutentur ordinatarum (y) ad ſecundas abſciſſas inclinationes; ut anguli quos cum ſuis ſecundis abſciſſis ſint reſpective inter ſe æquales; & ſupponantur valores utrarum abſciſſarum & earum correſpondentium ordinatarum reſpective æquales eſſe, & conſequenter trium incognitarum quantitatum (x, z, y) iidem ſunt valores in utriſque æquationibus, ergo duæ reſultant æquationes quæſitæ tres incognitas quantitates (x, z, y) habentes; quæ reduci poſſint in unam, ita ut exterminetur incognita quantitas vel x vel y vel z.

PROB. XXIII.

Fig. 45. Datis duabus æquationibus, quarum una exprimit relationem inter primam abſciſſam AP, & ejus ordinatas (PM) in eodem plano inſiſtentes; altera vero exprimit relationem inter ordinatam PM, abſciſſam AP, partes (Pp) prædictæ ordinatæ PM tanquam ſecundam abſciſſam & ordinatas pm per punctum abſciſſæ p tranſeuntes, invenire æquationem relationem inter abſciſſam Lp & ejus correſpondentes ordinatas pm per punctum p tranſeuntes, cujuſcunque ſectionis (LMN) dati ſolidi exprimentem.

1. Sectio LMN ſecet ſectionem mMn, &c. cujus abſciſſa fuit Lp & ordinata pm in lineâ pm quæ eſt ordinata ad ſecundam abſciſſam ~~per punctum L tranſeuns.~~

Scribantur pro ſinubus angulorum, quos ſectio facit cum axe AP (pLP), cum ejus ordinatis (LpP), & ordinatæ faciunt cum ſuo axe

(M.

(MPL), reſpectìve t, r, s; & ſubſtituatur pro diſtantiâ inter punctum L, in quo ſectio quæſita ſecat axem, & abſciſſæ axis verticem ($AL=a$;) & pro abſciſsâ Lp & ejus correſpondente ordinatâ pm ſectionis quæſitæ reſpective x & y; tum erit ſin. $\angle pPL\ (s)$: ſin. $\angle PpL\ (r) :: Lp\ (x)$: LP, & exinde $LP = \frac{rx}{s}$, & axis abſciſſa ($AP = AL + LP = \frac{rx}{s} + a$;) etiamque ſin. $\angle pPL\ (s)$: ſin. $\angle pLP\ (t) :: Lp\ (x) : Pp$, unde ſecunda abſciſſa $pP = \frac{tx}{s}$. Subſtituantur hæ quantitates $\left(\frac{rx}{s} + a \text{ \& } \frac{tx}{s}\right)$ pro ſuis valoribus (AP & Pp) in datis æquationibus, & reſultant duæ æquationes tres incognitas quantitates (x, y & PM) habentes, reducantur hæ duæ æquationes in unam, ita ut exterminetur incognita quantitas (PM); & reſultat æquatio relationem exprimens inter incognitas (x & y) quantitates abſciſſam & ejus correſpondentes ordinatas ſectionis quæſitæ.

Fig. 42. 2. Sectio LMN ſecet ſectionem mMn, &c. cujus abſciſſa fuit Pp & ordinata pm, haud in lineâ pm ordinatâ ad ſecundam abſciſſam, ſed in lineâ (pr,) prædictam ordinatam pm in dato angulo (mpr) ſecante.

Scribantur pro ſinubus angulorum $\angle Ppr$, $\angle mpr$, & $\angle Ppm$ reſpective l, m & n; & per coroll. 1. prob. 1. ita transformetur æquatio exprimens relationem inter abſciſſam Pp & ejus ordinatas pm, &c. ut exprimat relationem inter prædictam abſciſſam Pp & ejus ordinatas pr cum prædictis ordinatis pm angulum mpr facientes. Erit ſin. $\angle Ppm\ (n)$: ſin. $\angle Ppr\ (l) :: pr\ (v) : rh$, & exinde $\frac{lv}{n} = pm, = rh$, & per prædictum coroll. abſciſſa, cujus ordinata fuit pm (ſi modo abſciſſa, cujus ordinata fuit pr, ſit z) erit $\frac{mv}{n} + z = Pp$. Subſtituantur hæ quantitates $\left(\frac{lv}{n}, \frac{mv}{n} + z\right)$ pro ſuis valoribus pm & Pp in datâ

datâ æquatione: in æquationibus reſultantibus per præcedentem caſum inveniantur $z = \frac{tx}{s}$ & $AP = \frac{rx}{s} + a$; quibus quantitatibus $\left(\frac{tx}{s} \text{ \& } \frac{rx}{s} + a\right)$ pro ſuis valoribus (z & AP) in reſultantibus æquationibus ſubſtitutis, orientur æquationes deſideratæ; i. e. in datis æquationibus pro lineis pm, Pp & Ap ſubſtituantur reſpective $\frac{lv}{n}$, $\frac{mv}{n} + \frac{tx}{s}$ & $\frac{rx}{s} + a$; exorientur duæ æquationes deſideratæ relationem inter PM, abſciſſam Lp, & ejus correſpondentes ordinatas exprimentes: reducantur hæ duæ æquationes in unam, ita ut exterminetur incognita quantitas PM; reſultat æquatio relationem exprimens inter incognitas quantitates x & v abſciſſam & ejus correſpondentes ordinatas ſectionis quæſitæ.

Cor. 1. Hujus ſolidi ſectio non poteſt eſſe curva majorum dimenſionum, quam quæ continentur in rectangulo ſub dimenſionibus duarum datarum æquationum in ſeſe ductis.

Fig. 45. 3. Datâ æquatione exprimente relationem inter abſciſſam AP, ſecundam abſciſſam Pp, & ejus correſpondentes ordinatas pm; in datâ æquatione pro quantitatibus AP, Pp & pm ſubſtituantur earum prius deducti valores $\frac{rx}{s} + a$, $\frac{mv}{n} + \frac{tx}{s}$, & $\frac{lv}{n}$ (ubi literæ r, s, a, m, n, t, l; x & v eaſdem, quas in precedenti ~~problemate~~ Casu, quantitates denotant); reſultat æquatio relationem inter abſciſſam x & ejus correſpondentes ordinatas v ſectionis quæſitæ exprimens.

Cor. 1. Præcedens caſus in hunc facile reduci poſſit; duæ enim æquationes, relationes inter lineas AP, Pp, pm & PM exprimentes, e vulgaris algebræ principiis reducantur in unam, ita ut exterminetur incognita quantitas PM; reſultabit æquatio prædicta relationem inter incognitas quantitates AP, Pp & Pm exprimens.

Cor. 2. Dimenſiones reſultantis æquationis nunquam majores eſſe poſſunt quam datæ æquationis dimenſiones.

Ex principiis conſimilibus transformari poſſunt prima & ſecunda abſciſſa & ejus ordinatæ, ſolidi, cujus æquatio datur, in quaſcunque alias, & exinde æquationes ad quaſcunque ſolidi prædicti ſectiones deduci poſſunt.

Cor. 3. Datâ æquatione ad ſolidum per coroll. 1. prob. 8. inveniri poſſint æquationes ad ſimilia ſolida, &c.

THEOR. XIV.

Fig. 45. Datis duabus æquationibus relationem exprimentibus inter abſciſſam *AP* ſecundam abſciſſam *Pp* & ejus correſpondentes ordinatas *p m*, & tres incognitas quantitates habentibus; hæ erunt æquationes ad curvam algebraicam duplicis curvaturæ; i. e. curvam, cujus puncta haud collocantur in eodem plano.

Cor. 1. E principiis prius traditis pro transformatione abſciſſarum & ordinatarum ſimplicis curvaturæ curvarum facile transformari poſſunt datæ abſciſſæ & ordinatæ curvarum duplicis curvaturæ in alias, quaſcunque habentes ad priores inclinationes.

Et ſic de inveniendis diametris, aſymptotis, &c.

Cor. 2. Subtangens ad curvam duplicis curvaturæ duci poteſt eodem fere modo, quo in algebraicis curvis, &c.

PROB. XXIV.

Sint æquationes datæ ad duas duplicis curvaturæ curvas; invenire earum interſectiones.

Ita transformentur abſciſſæ duarum curvarum & earum ordinatarum inclinationes, ut evadant eædem in unâ curvâ ac in alterâ.

Ob datas æquationes ad duas curvas duplicis curvaturæ; i. e. datas quatuor æquationes tres incognitas quantitates habentes; e reductione

tione harum æquationum ita ut exterminentur incognitæ quantitates, facile conſtant interſectiones, ſi modo poſſint eſſe ullæ.

Hic adjici poſſunt propoſitiones ad algebraica ſolida & curvas duplicis curvaturæ, quæ conſimiles ſunt fere omnibus propoſitionibus capite primo traditis de curvis ſimplicis curvaturæ: ſed tædet has diſquiſitiones ulterius promovere.

PROB. XXV.

Fig. 46. Sit curva *M M*, &c. cujus æquatio relationem inter abſciſſam ($P'p' = x$) & ejus correſpondentes ordinatas ($p'm' = y$) per punctum *p* tranſeuntes exprimens datur, & datur etiam punctum lucidum *A*, invenire projectionem datæ curvæ a puncto lucido *A* in datum planum *L p M*.

Ducatur conus, cujus vertex eſt punctum lucidum *A* & baſis data curva, & producatur conus; hujus coni parallelæ ſectiones erunt inter ſe ſimiles, & conſequenter per cor. 1. prob. 5. in datâ æquatione pro incognitis quantitatibus x & y ſcribantur reſpective $\frac{AP'}{AP}z$ & $\frac{AP'}{AP}v$, & reſultat æquatio relationem exprimens inter abſciſſam z & ejus ordinatas v ſectionis $M' P M'$, &c. ſolidi datæ curvæ parallelæ. 1. planum *LMN* ſecet umbram vel ſectionem *MmM*, &c. quæ datæ curvæ eſt parallela & ſimilis in lineâ *p m*, quæ eſt ordinata ad abſciſſam $P'p$, i. e. projectionem datæ curvæ abſciſſæ $P'p'$, ſit ſin. $\angle LPp\ (s)$: ſin. $\angle PpL\ (r) :: pL\ (X) : PL$; ergo $PL = \frac{rX}{s}$, & $PL + LA = \frac{rX}{s} + a = AP$; ſit etiam ſin. $\angle LPp\ (s)$: ſin. $\angle PLp\ (t) :: pL\ (X) : Pp$; & conſequenter abſciſſa $Pp = \frac{tX}{s}$; ſubſtituantur hæ quantitates $\frac{rX}{s} + a$ & $\frac{tX}{s}$ pro ſuis valoribus AP & $Pp = z$ in prædictâ reſultanti æquatione, & exinde oritur æquatio relationem inter abſciſſam Lp & ejus correſpondentes ordinatas pm quæſitæ ſectionis deſignans, i. e. in

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in datâ æquatione ſubſtituantur pro incognitis quantitatibus x & y earum valores $\frac{AP' \times t \times X}{rX + as}$ & $\frac{AP' \times v}{\frac{rX}{s} + a}$, & reſultat æquatio relationem inter X & v abſciſſam & ordinatam quæſitæ ſectionis exprimens.

Fig. 77. 2. Sectio LMN ſecet ſectionem mMn, &c. cujus abſciſſa fuit Pp & ordinata pm haud in lineâ pm ordinatâ ad abſciſſam Pp, ſed in lineâ pr ~~vel Lr~~ prædictam ordinatam pm in dato angulo (mpr) ſecante: ſcribantur pro ſinubus angulorum Ppr, mpr, & Ppm reſpective l, m & n; & per cor. 1. prob. 1. ita transformetur data æquatio ut exprimat relationem inter prædictam abſciſſam Pp & ejus ordinatas pr cum prædictis ordinatis pm angulum mpr facientes. Erit ſin. $\angle Ppm\,(n)$: ſin. $\angle Ppr\,(l)$ :: $pr\,(V) : pm$, & exinde $\frac{lV}{n} = pm$, & per prædict. coroll. 1. abſciſſa, cujus ordinata fuit pm (ſi modo abſciſſa, cujus ordinata fuit pr, ſit Z) erit $\frac{mV}{n} + Z = Pp$. Subſtituantur hæ quantitates $\left(\frac{lV}{n}\right.$ & $\left.\frac{mV}{n} + Z\right)$ pro ſuis valoribus $= y$ & $Pp = x$, in data æquatione, & reducitur hic caſus in præcedentem: ſubſtituantur igitur in reſultante æquatione pro incognitis quantitatibus Z & V reſpective $\frac{AP' \times t \times X}{rX + as}$ & $\frac{AP' \times v}{\frac{rX}{s} + a}$, & reſultat æquatio quæſita: i. e. pro x & y in datâ æquatione ſcribantur reſpective $\frac{m \times AP' \times v}{\frac{nrX}{s} + na} + \frac{AP' \times t \times X}{rX + as}$, & $\frac{l \times AP' \times v}{\frac{nrX}{s} + na}$ & reſultat æquatio relationem inter X & v abſciſſam & ordinatam quæſitæ ſectionis exprimens.

Cor. 1. Hinc nulla curva projiciet curvas ſuperiorum ~~ſibi ipſi~~ ordinum. *is quam ipsa habet*

Cor.

Cor. 2. Parallelæ ſectiones projectæ ſemper erunt ſimiles curvæ.

Cor. 3. Hinc inveniri poſſint quæcunque curvæ, quæ projicere poſſint datas curvas; ſint enim v & X abſciſſa & ordinata datæ curvæ; ſupponantur $\frac{m \times AP' \times v}{\frac{nrX}{s} + na} + \frac{AP' \times t \times X}{rX + as} = x$ & $\frac{l \times AP' \times v}{\frac{nrX}{s} + na} = y$; unde $v = \frac{tnay}{mry - lrx + l \times AP' \times t}$ & $X = \frac{aslx - asmy}{mry - lrx + l \times AP' \times t}$; quibus quantitatibus pro ſuis valoribus in æquatione ad datam curvam ſubſtitutis, reſultat generalis æquatio ad omnes curvas quæſitas.

Hinc facile colligi poſſit curvas, quarum æquationes ſint $y^{2n} + \overline{c + dx + ex^2}\, y^{2n-1} + \&c. = 0$, projicere poſſe omnes curvas $(2n+1)$ dimenſionum.

Et ſic e fluxionum principiis ratiocinari liceat de fluxionalibus curvis.

PROB. XXVI.

Fig. 47. Dato puncto lucido Q, & duabus æquationibus; quarum una exprimit relationem inter abſciſſam AP & ejus correſpondentes ordinatas (PM) in eodem plano; altera vero relationem inter ordinatam PM, abſciſſam AP, partes (Pp) ordinatæ (PM) tanquam ſecundam abſciſſam, & ejus ordinatas pm per punctum p ſecundæ abſciſſæ tranſeuntes; invenire ejus projectionem in datum planum.

Ita transformentur hæ duæ æquationes in unam, ut exterminetur incognita quantitas PM; reſultans æquatio exprimet relationem inter abſciſſam AP, ſecundam abſciſſam Pp, & ejus correſpondentes ordinatas pm; i. e. erit æquatio ad ſolidum.

Deinde per prob. 1. ita transformetur ſecunda abſciſſa (Pp), ut ejus planum fiat idem ac planum linearum jungentium puncta primæ abſciſſæ (AP) & punctum lucidum; vel quod idem eſt, ducatur a puncto

puncto lucido Q ad primam abſciſſam AP linea QH parallela plano datarum ſectionum Ppm; & ita transformetur ſecunda abſciſſa (Pp) datæ ſectionis Ppm, ut ſecunda abſciſſa Pp ſemper ſit in eodem plano cum linea QH.

Tertio tranſeat datum planum per primam abſciſſam AP in directione ordinatis (pm) ad ſecundam abſciſſam parallelâ.

Ductâ lineâ QH parallelâ ſecundis abſciſſis (Pp), & primæ abſciſſæ (AP) in puncto H occurrente; ſit L quodlibet abſciſſæ AP punctum, ducatur linea QL, & ad partes lineæ QL trahantur correſpondentes ordinatæ $(pm$, &c.) ducatur etiam linea Qm tangens ſolidum prædictum in (m), & ſecans datum planum per abſciſſam AP tranſeuns in puncto o; & linea (Pp) ſecunda abſciſſa ad ſolidum, i. e. abſciſſa, cujus correſpondens ordinata ſit pm: ducatur etiam a puncto Q perpendiculum (Qb) in primam abſciſſam AP; ſcribantur pro lineis QH, AH, Qb, bH, HL, Lo, Lp, pm, Pp, PA reſpective a, b, c, d, x, y, X, v, z, w: in æquatione ad ſolidum deductâ ſubſtituantur pro primâ & ſecundâ abſciſsâ $(AP = w$ & $Pp = z)$ earum reſpectivi valores $x\ (HL) + b\ (AH) - \frac{xX\,(Lp)}{\sqrt{c^2 + \overline{x-d}^2} = QL}$, & $\frac{aX}{\sqrt{c^2 + \overline{x-d}^2}}$ (qui valores facile deduci poſſint e ſimilibus triangulis LQH & LpP); reſultat æquatio relationem inter abſciſſam $Lp\ (X)$ & ejus ordinatas $pm\ (v)$ exprimens: inveniatur ſubtangens hujus curvæ, quæ ſiat æqualis quantitati $\sqrt{c^2 + \overline{x-d}^2}\ (QL) - X\,(Lp)$; & in æquatione reſultanti pro $pm = v$ ſubſtituatur ejus valor $y - \frac{yX}{\sqrt{c^2 + \overline{x-d}^2}}$ (qui conſtat e ſimilibus triangulis LQo & pQm); reſultant duæ æquationes tres incognitas quantitates $(x, y$ & $X)$ habentes, quibus in unam reductis, ita ut exterminetur incognita quantitas (X); reſultat æquatio relationem inter abſciſſam $(x = HL)$ & ejus correſpondentes ordinatas $y\ (Lo)$ ſolidi projectionis in datum planum $(LoH$, &c.$)$ exprimens.

Ex.

Ex. 1. Sit ſolidum cylindricum, cujus baſis ſit quæcunque curva, & ejus laterum projectiones in datum planum erunt rectæ lineæ.

Ex. 2. Sit ſolidum conus, cujus baſis eſt quæcunque curva, & ejus laterum projectiones erunt rectæ lineæ.

Facile conſtant hæc etiam e vulgari trigonometriâ & doctrinâ fluxionum.

Cor. 1. Si vero datum planum (*A o*, &c.) haud tranſit per primam abſciſſam *AP* & haud eandem inclinationem ad abſciſſam *AP* habet, quam habent ordinatæ *pm*; e projectione in planum per abſciſſam *AP* tranſeuns & puncto lucido *Q* per prob. 25. inveniri poſſit projectio in quodcunque aliud planum.

Cor. 2. Sit æquatio ad ſolidum *n* dimenſionum, & æquatio ad curvam projectam haud majores habet dimenſiones quam numerus punctorum, in quibus lineæ a dato puncto datam curvam *n* dimenſionum tangentes duci poſſunt.

PROB. XXVII.

Fig. 48. Datis puncto lucido *Q* & duabus æquationibus, ut in problematibus præcedentibus, relationem inter primam abſciſſam *AP* & *PM*, & inter primam *AP* & ſecundam abſciſſam *Pp* & ejus correſpondentes ordinatas *pm* & *PM*; & tertiâ æquatione relationem inter abſciſſam ($\alpha\,\pi$) & ejus correſpondentes ordinatas ($\pi\,\mu$) datæ curvæ exprimente, invenire projectionem datæ curvæ in datum ſolidum.

Ita transformentur duæ priores æquationes in unam, ut exterminetur incognita quantitas *PM*; & reſultans æquatio exprimit relationem inter abſciſſam *AP* ſecundam abſciſſam *Pp* & ejus correſpondentes ordinatas *pm*, i. e. erit æquatio ad ſolidum.

Transformetur etiam abſciſſa curvæ vel prima abſciſſa ſolidi, ita ut fiat projectio abſciſſæ curvæ in primam dati ſolidi abſciſſam; & ita transformetur ſecunda abſciſſa, ut ſit in eodem plano cum abſciſsâ curvæ & primâ ſolidi abſciſsâ; & transformentur ſecundæ abſciſſæ ordinatæ,

ordinatæ, $(p\,m)$ ita ut fiant parallelæ plano datæ curvæ, vel ordinatis $(\pi\,\mu)$ datæ curvæ.

Ducatur linea QH parallela ſecundæ abſciſſæ $(P\,p)$ & occurrens abſciſſæ curvæ in r, & primæ ſolidi abſciſſæ in H.

Ducatur per punctum curvæ (μ) linea $(Q\mu)$ occurrens ſolido in puncto m, & per correſpondens ejus abſciſſæ punctum (π) ducatur linea Qp.

Scribantur pro lineis Qr, QH, αr, HA, AP, Pp, pm, $\alpha\pi$, πm reſpective $a, b, c, d, x, z, v, X, Y$; tum propter duo ſimilia triangula $Q\pi\mu$ & Qpm erit $Qr : QH - Pp :: \pi\mu : pm$, i. e. $a : b - z :: Y : v$ & eodem modo deducitur ſubſequens proportio $Qr : QH - Pp :: \alpha\pi - \alpha r : AP - AH$, i. e. $a : b - z :: X - c : x - d$; e primâ proportione ſequitur $Y = \frac{av}{b-z}$, e ſecundâ vero $X = \frac{ax - ad + bc - zc}{b - z}$, quibus quantitatibus pro ſuis valoribus in æquatione relationem inter abſciſſam curvæ (X) & ejus ordinatas (Y) exprimente reſultant duæ æquationes tres incognitas quantitates (x, z, v) habentes, quibus in unam reductis ita ut exterminetur una incognita quantitas, reſultat æquatio quæſita relationem inter duas reliquas exprimens.

Cor. 1. Si vero prima ſolidi abſciſſa haud ſit projectio abſciſſæ curvæ, & ordinatæ ſecundæ abſciſſæ ſolidi ordinatis datæ curvæ ~~haud~~ ſint parallelæ; & ſecunda abſciſſa ſolidi ~~haud~~ ſit in eodem plano cum primâ ſolidi & curvæ abſciſsâ, tum ex hoc problemate & prob. 1mo. petenda eſt ſolutio.

Fig. 49. Cor. 2. Solidi illuminati contentum inveniri poteſt e principiis fluxionum, inveſtigando aream curvæ contentam inter lineas $(m\,p\,m)$ & arcum curvæ $mRSm$, & ducendo hanc aream in fluxionem abſciſſæ AP; & erit reſultans ſolidum fluxio fruſti ſolidi quæſiti.

Cor. 3. Arcûs curvæ umbram terminantis longitudo inveniri poteſt inveſtigando fluentem fluxionis $\sqrt{\dot{z}^2 + \dot{w}^2 + \dot{v}^2}$, in quâ $\dot{z}$, $\dot{w}$, $\dot{v}$ reſpective denotant fluxiones primæ abſciſſæ, ſecundæ abſciſſæ & ejus ordinatæ; ſi modo prima abſciſſa cum ſecundâ, & ſecunda cum ejus ordinatis rectum angulum faciant.

Ex

Ex iiſdem etiam principiis inveniri poſſunt generaliter ſolidorum ſuperficies.

Cor. 4. Dimenſiones æquationis reſultantis non plures eſſe poſſunt quam rectangulum ſub dimenſionibus æquationis ſolidi & datâ curvæ æquatione.

Hinc inveniri poſſit projectio umbræ dati ſolidi in datum ſolidum: inveniri enim per propoſitionem præcedentem poſſit projectio ſolidi in planum, & per hanc propoſitionem projectio plani in ſolidum, & exinde inveniri poſſit projectio ſolidi in ſolidum.

Si vero loco puncti lucidi *Q* ſupponatur quæcunque ſuperficies vel ſolidum, facile conſtat methodus projectionem ſolidi vel ſuperficiei inveniendi in datum planum vel ſolidum.

Datâ æquatione ad ſolidum exprimente relationem inter prædictas abſciſſas & ordinatas, facile e trigonometriâ & vulgari algebrâ deduci poſſit æquatio ad ſolidum relationem inter lineas e quibuſcunque datis punctis ductas exprimens.

Hæc omnia facile ad fluxionales curvas & ſolida applicari poſſint.

E principiis hujuſce generis inveniri poſſit, utrum una curva per aliam tranſire poſſit, necne.

Hìc inſerere potui curvoidalem trigonometriam, quæ in ſuo calculo multum habet laborem, ſed ejus principia ſatis perſpicua ſunt, etiamque multæ aliæ propoſitiones de ſolidis colligi poſſunt, ſed ſatis quoad noſtrum propoſitum de his dictum eſt.

CAP. IV.

Curvarum Proprietates.

LEMMATA.

Fig. 51. I. Dividatur ſingulum trianguli (EFG) latus in duas æquales partes $EA = AG$, $GB = BF$, $FC = CE$; jungantur puncta biſectionum A, B, C; & erit circumſcriptum triangulum ad inſcriptum in ratione numeri quaternarii ad unitatem.

Fig. 52. 2. Dividatur ſingulum latus trapezii $EFGH$ in duas æquales partes $EA = AF$, $FB = BG$, $GC = CH$, $HD = DE$; jungantur puncta biſectionum $ABCDA$, & circumſcriptum trapezium erit ad inſcriptum in ratione numeri binarii ad unitatem.

Dem. Ducantur diagonales EG, FH, & ex hypotheſi $AE = AF$, & $FB = BG$, & conſequenter triangulum (AFB) erit quarta pars trianguli (FEG); & ſic triangulum (DHC) erit quarta pars trianguli $(HEG,)$ & ſumma duorum triangulorum $FAB + DHC$ erit quarta pars dati trapezii $(EFGH)$, & ſic ſumma duorum triangulorum $(AED + BGC)$ erit quarta pars dati trapezii; ergo ſumma quatuor triangulorum $(AFB + BGC + CHD + DEA)$ erit dimidium dati trapezii; ergo ſumma quatuor triangulorum $(AFB + BGC + CHD + DEA)$ & exinde trapezium $(ABCD)$ erit ad datum trapezium $(EFGH)$ in ratione unitatis ad numerum binarium.

Cor. Figura inſcripta $(ABCD)$ erit parallelogrammum.

In omnibus polygonis hâc methodo inſcriptis area polygoni inſcripti erit ad aream dati polygoni circumſcripti in majori ratione quam unitatis ad numerum binarium.

Fig. 53. 3. Dato quocunque trapezio $(ABCD)$, cujus diagonales ſunt lineæ $(AC$ & $BD)$, per puncta (A, B, C, D) ducantur lineæ diagonalibus $(AC$ & $BD)$ parallelæ; & erit figura $(EFGH)$ parallelogrammum.

Cor.

Cor. Circumſcriptum parallelogrammum ($EFGH$) duplum eſt dati trapezii ($ABCD$); parallelogrammum enim AG duplum eſt trianguli (ABC), & parallelogrammum AH duplum eſt trianguli (DAC), ergo totum parallelogrammum (EG) duplum eſt trapezii ($ABCD$).

Fig. 54. Sit polygonum $ABCD$, &c. ducantur diagonales AC, BD, CD', &c. per punctum B ducatur linea FE parallela lineæ AC; per (C) ducatur linea GF parallela lineæ BD; per (D) ducatur linea (HG) parallela lineæ CD', & ſic deinceps; & polygonum circumſcriptum non habet ad datum polygonum majorem rationem quam numerus binarius ad unitatem.

4. Sit polygonum $EFGHI$, &c. ducantur lineæ AB, BC, CD, DD', &c. ita ut angulus ABE æqualis ſit angulo CBF; angulus BCF æqualis ſit angulo DCG; $\angle CDG = \angle D'DH$, &c. ſit primo numerus polygoni laterum impar, & ſcribantur pro angulis dati polygoni EFG, FGH, GHI, &c. reſpective a, b, c, d, e, &c. tum erit angulus $CBF = \frac{180 - a + b - c + d - e + \&c.}{2}$ angulus $BCF = \frac{180 - b + c - d + e - \&c.}{2}$ &c.

Demonſtratio. Scribantur pro angulis CBF, BCF, CDG, $DD'H$, $D'D''I$, &c. reſpective x, y, z, v, w, &c. & reſultant æquationes $a + x + y = 180°$, $b + z + y = 180°$, $c + v + z = 180°$, $d + v + w = 180°$, $e + x + w = 180°$, &c. &c. ita reducantur hæ æquationes in unam, ut exterminentur omnes incognitæ quantitates præter unam, quæ ſit (x), & fit $x = \frac{180 - a + b - c + d - \&c.}{2}$, ſi quicunque angulus inventus ſit negativus, tum tale polygonum inſcribi nequit.

2. Sit numerus polygoni laterum par, & hujuſce generis polygonum inſcribi nequit, ni $a - b + c - d + \&c. = 0$.

Cor. 3. Scribantur pro lineis EF, FG, GH, HI, IK, &c. reſpective P, Q, R, S, T, &c. pro lineâ EA ſcribatur (x), ſit $EA : EB :: 1 : o$; $BF : FC :: 1 : n$; $CG : GD :: 1 : m$; $DH : HD' :: 1 : l$; &c.

tum $EB = ox$, $FC = n \times \overline{EF - EB} = nP - nox$, $GD = m \times \overline{FG - FC} = mQ - mnP + mnox$, $HD' = l \times \overline{GH - GD} = lR - lmQ + lmnP - lmnox$, & ſic deinceps. Et exinde e ſimplici æquatione reſultante eruitur valor incognitæ quantitatis (x), quo ſubſtituto in prædictis æquationibus reſultant reliquarum incognitarum quantitatum (y, z, v, &c.) valores.

Et ſic de quamplurimis hujuſmodi propoſitionibus.

PROB. XXIV.

Invenire proprietates curvarum, quarum nulla datur æquatio relationem inter abſciſſam & ejus correſpondentes ordinatas deſignans.

Proprietates ejuſdem vel ad unum ſolummodo curvæ punctum, vel ad plura, vel ad omne punctum referuntur.

1. Si ad unum ſolummodo punctum referuntur lineæ, inter quas requiruntur æquationes relationem exprimentes, tum proprietates e vulgaris analyſeos, geometriæ & incrementorum principiis petendæ ſunt, & in omnibus curvis vel algebraicis vel fluxionalibus, &c. eodem modo deducendæ.

Fig. 55. Ex. Ita transformare æquationem fluxionalem, in quâ continentur duæ variabiles quantitates (x & y), quarum x fluat uniformiter, ut fluat uniformiter (y).

Sit quæcunque curva (MRS), cujus abſciſſa ($x = AP$) fluit uniformiter, ſit ejus fluxio (Pp), & ſit fluxio ordinatæ ($y = PM$) Mh; ſecunda vero ejus fluxio $2mf$; nunc fluat uniformiter ordinata (PM), ſit ejus fluxio (Mh), tum prima fluxio abſciſſæ (AP) erit he, & ſecunda ejus fluxio $2me$; & duo triangula Mhe & fme ſunt ſimilia; ergo eh ($\dot{x}$) : hM ($\dot{y}$) :: $2em$ ($\ddot{x}$) ſecunda fluxio abſciſſæ (x), cum fluat uniformiter ordinata (y) : $2mf$ ($\ddot{y}$) ſecundam ordinatæ fluxionem, cum fluat uniformiter abſciſſa (x), & conſequenter $\ddot{y} = \frac{-\dot{y}\ddot{x}}{\dot{x}}$ quo valore pro ($\ddot{y}$) in datâ fluxionali æquatione ſubſtituto, & ejus fluxione pro ($\ddot{y}$), &

& ſic deinceps; & ita transformabitur æquatio, ut fluat uniformiter (y).

Eodem modo deduci poſſint methodi tangentes, ſubnormales, radios curvaturæ, aſymptotos, &c. inveniendi.

Et ſic de omni hujuſce generis transformatione.

2. Si vero ad plura puncta referuntur lineæ, inter quas requiruntur æquationes relationem exprimentes, tum data relatio vel exprimitur per algebraicas vel fluxionales æquationes, &c. ſi vero per algebraicas æquationes, tum e vulgaris analyſeos & geometriæ principiis petenda eſt ſolutio.

Si vero per fluxionales æquationes exprimatur data relatio, tum inveniendæ ſunt lineæ, quæ inter ſe eaſdem habent rationes, quas habent inter ſe fluxiones datarum æquationum, quibus pro fluxionibus reſpective ſubſtitutis, transformabuntur fluxionales æquationes in algebraicas, & ex iiſdem principiis fluxionales ac algebraicæ ſolutionem recipiant.

Sequuntur hujus casûs quædam exempla, quæ præcipue de quantitatibus cum maximæ vel minimæ fiunt, verſantur: i. e. cum incrementa quantitatum evaneſcant vel nihilo fiant æqualia; hoc enim in caſu quantitates vel maximæ vel minimæ fiunt vel contrariam flexuram habent.

Fig. 56. Ex. 1. Sit ovalis nullum habens contrariæ flexuræ punctum, & ducantur (n) lineæ (EF, GH, HI, &c.) ovalem tangentes in n punctis (A, B, C, D, &c.) ſint etiam hæ lineæ ad contactuum puncta (A, B, C, D, &c.) in duas æquales partes diviſæ, i. e. $EA = AF$, $FB = BG$, $GC = CH$, $HD = DI$, &c. tum minus erit polygonum ($EFGH$, &c.) quam quodvis aliud ejuſdem numeri laterum, & idem punctum contactûs (A) habens, & circa datam ovalem deſcriptum.

Si enim circumſcriptum polygonum, cujus latera EF, FG, GH, &c. in duas æquales partes in punctis contactuum (A, B, C, D, &c.) dividuntur, non ſit minimum, quod circa ovalem deſcribi poteſt, & cujus punctum contactûs eſt A, ſit polygonum ($LMNR$ &c.) minimum, habens latus (MN) a puncto contactûs (a) in duas inæquales partes aM & aN diviſum, quarum aM major ſit quam aN.

Ducatur

Ducatur linea $\alpha\beta$ lineas LM, NR & MN in punctis α, β & b reſpective ſecans, curvam vero tangens in puncto γ ad punctum a ita appropinquante, ut linea bM major ſit quam bN, & αb quam $b\beta$; tum polygonum $L\alpha\beta NR$, &c. minus erit quam polygonum $LMNR$, &c. quoniam triangulum $Mb\alpha$ majus eſt quam triangulum $Nb\beta$; & conſequenter polygonum ($LMNR$, &c.) non eſt minimum, quod circumſcribi poteſt, niſi ſingulum ejus latus in duas æquales partes a puncto ejus contactûs dividitur.

Fig. 59. Cor. 1. Sit polygonum triangulum EFG, & jungantur puncta contactuum, A, B, C; & erit triangulum inſcriptum ABC ad circumſcriptum EFG :: 1 : 4.

Fig. 60. Cor. 2. Sit polygonum trapezium $EFGH$, jungantur puncta contactuum A, B, C, D, & erit figura $ABCD$ parallelogrammum, & dimidium trapezii $EFGH$; & ſic de polygonis.

Fig. 57. Ex. 2. Sit ovalis ſemper concava in ſe, & in eâ inſcribatur polygonum n laterum ($ABCD$, &c.) & per ſucceſſiva puncta (A, B, C, D, &c.) ducantur tangentes (EF, FG, GH, &c.) reſpective, & ſit linea AC parallela lineæ tangenti ovalem in puncto B, linea BD parallela lineæ ovalem tangenti in puncto C, & ſic deinceps; tum polygonum $ABCD$, &c. erit maximum polygonum n laterum, quod in datâ ovali inſcribi poteſt, & cujus punctum contactûs eſt A.

Si enim lineæ AC, BD, CD', &c. non ſunt reſpective parallelæ lineis ovalem tangentibus in punctis (A, B, C, D, &c.) ſit BD haud parallela tangenti in puncto C, & lineæ BD ducatur linea $\alpha\gamma$ parallela, & curvam tangens in puncto α: tum triangulum $B\alpha D$ majus erit quam triangulum BCD; & conſequenter polygonum $ABCD$, &c. non eſt maximum polygonum n laterum, quod in datâ ovali inſcribi poteſt, habens ejus verticem punctum A, ni prædictæ lineæ ſint parallelæ reſpectivis tangentibus.

Fig. 59. Cor. 1. Sit linea AC parallela tangenti GF, BA parallela tangenti GE, & BC tangenti EF parallela; & erit ABC maximum inſcriptum triangulum, & EFG minimum circumſcriptum, quorum punctum contactûs eſt A; & conſequenter per lemma primum hujus capitis minimum circumſcriptum triangulum, cujus punctum contactûs

tactûs eſt *A* erit ad maximum inſcriptum ejuſdem puncti contactûs *A* :: 4 : 1.

Fig: 60. Cor. 2. Sit trapezium *ABCD* maximum, quod in datâ ovali inſcribi poſſit, & ad puncta ejus contactuum (*A*, *B*, *C*, *D*) ducantur tangentes *EF*, *FG*, *GH*, *HE*; & erit figura *EFGH* parallelogrammum, & parallelogrammum circumſcriptum *EFGH* ad inſcriptum trapezium *ABCD* :: 2 : 1.

Fig. 57. Ex. 3. Iiſdem poſitis inſcribatur polygonum *n* laterum (*ABCD*, &c.) & per ſucceſſiva puncta (*ABCD*, &c.) ducantur lineæ *EF*, *FG*, *GH*, *HI*, &c. ovalem tangentes in punctis (*A*, *B*, *C*, *D*, &c.) & ſit angulus $ABF = \angle CBG$, & angulus $BCG = \angle DCH$, & $\angle CDH = \angle D'DI$, &c. i. e. anguli, quos latera polygoni utrinque cum reſpectivis tangentibus ad contactuum puncta faciunt, ſint inter ſe æquales. Et ſumma polygoni (*n*) laterum, viz. $AB + BC + CD +$ &c. major erit quam ſumma cujuſcunque alterius polygoni *n* laterum in datâ ovali inſcripti, & cujus vertex eſt punctum *A*.

Fig. 57. Ex. 4. Deſcribatur circa ovalem in ſe concavam polygonum *n* laterum (*EFGH*, &c.) cujus punctum contactûs eſt *A*, ſint puncta contactuum reſpective *A*, *B*, *C*, *D*, &c. & ſit *AE* : *AF* :: tang. + ſec. comp. $\angle AFG$: tang. + ſec. comp. $\angle AEP$ . . *FB* : *BG* :: tang. + ſec. comp. $\angle BGC$: tang. + ſec. comp. $\angle BFA$ . . *GC* : *CH* :: tang. + ſec. comp. $\angle CHD$: tang. + ſec. comp. $\angle CGB$, & ſic deinceps, tum ſumma *n* laterum polygoni minor erit quam ſumma *n* laterum cujuſcunque alterius polygoni circa ovalem deſcripti, & cujus punctum contactûs eſt *A*.

Fig. 57. Cor. Inſcribatur in quâcunque ovali, quæ nullum habet contrariæ flexuræ punctum, polygonum *n* laterum (*ABCD*, &c.) cujus area vel ſumma *n* laterum ſit maxima; & ad puncta (*A*, *B*, *C*, *D*, &c.) ~~horum polygonorum~~ contactuum ducantur tangentes *EF*, *FG*, *GH*, &c. & erit contentum $EA \times FB \times GC \times HD \times$ &c. $= AF \times BG \times CH \times$ &c. vel circumſcribatur circa ovalem polygonum *n* laterum, cujus area vel ſumma *n* laterum ſit minima, & iterum

contentum

contentum $EA \times FB \times GC \times$ &c. $= AF \times BG \times CH \times$ &c. & vice verſâ, ſi utcunque ducatur polygonum n laterum circa prædictam ovalem, contentum $EA \times FB \times GC \times$ &c. $= AF \times BG \times CH \times$ &c. tum ad ſingulum ovalis punctum ſemper duci poſſit polygonum n laterum, cujus area vel ſumma n laterum inſcripti polygoni ſit maxima, circumſcripti vero polygoni ſit minima.

Fig. 57. 2. Ex. 5. Deſcribatur circa ovalem prædicti generis polygonum n laterum ($EFGH$, &c.) cujus puncta contactuum ſint (A, B, C, D, &c.) in ſingulam tangentem ducantur duo perpendicula a duobus proximis angulis contactûs puncto utrinque jacentibus; i. e. ducantur AP & CQ perpendicula in tangentem EBF; BR & DS perpendicula in tangentem FCG; & ſic deinceps: ſit etiam $AB^{m-2} : BC^{m-2} :: BQ : BP$, & $BC^{m-2} : DC^{m-2} :: CS : CR$, & ſic deinceps: tum ſumma $AB^m + BC^m + CD^m + DB^m +$ &c. major erit quam ſumma m poteſtatis e ſingulis n lateribus cujuſcunque alterius polygoni in ovali inſcripti, cujus punctum contactûs eſt A.

Ex. 6. Iiſdem lineis ductis, ſit $AB^2 : BC^2 :: M \times PB : o \times BQ$, & $BC^2 : CD^2 :: o \times CR : p \times CS$, & ſic deinceps; tum contentum $AB^m \times BC^o \times CD^p \times$ &c. majus erit quam quodcunque aliud hujuſce generis contentum, cujus punctum contactûs eſt A.

Fig: 57. Ex. 7. Iiſdem lineis ductis, invenire in quibus caſibus ſumma $EF^m + FG^m + GH^m + HI^m +$ &c. ſit minima quantitas, vel aggregatum e ſingulis contentis $EF^m \times FG^o \times GH^p \times$ &c. &c. ſit minima quantitas, vel quæcunque algebraica functio linearum EF, FG, GH, &c. ſit maxima vel minima quantitas; ſupponantur omnes lineæ (HI, &c.) præter tres ſucceſſivas lineas EF, FG, GH eſſe invariabiles, & inveniantur per vulgaria incrementorum principia fluxiones prædictarum quantitatum, viz. $m\,EF^{m-1}\,\dot{E}F + m \times FG^{m-1}\,\dot{F}G + m\,GH^{m-1}\,\dot{G}H$, & $EF^{m-1} \times FG^{o-1} \times GH^{p-1} \times \overline{m\,FG \times GH \times \dot{E}F + o \times EF \times GH \times \dot{F}G + p \times EF \times FG \times \dot{G}H}$, & fluxio datæ algebraicæ functionis; ſed hæ fluxiones nihilo evadent æquales, cum quantitates ipſæ fiant maximæ vel minimæ; fiant igitur nihilo æquales, & in reſultantibus æquationibus pro $\dot{E}F$, $\dot{F}G$, $\dot{G}H$ ſcribantur

tur

tur reſpective $BF \times$ ſec. comp. $< GFE$, $BF \times$ tang. comp. $< GFE - BG \times$ tang. comp. $< FGH$, $BG \times$ ſec. comp. $< FGH$, & reſultant æquationes deſideratæ.

Ex. 8. Si data quantitas, quæ requiratur eſſe maxima vel minima, ſit fluxionalis quantitas; tum ejus fluxio neceſſario involvet fluxiones ſuperiorum ordinum, fiat hæc fluxio nihilo æqualis, & inveniantur lineæ, quæ eaſdem inter ſe habent rationes, quas habent prædictæ fluxiones, quibus pro ſuis valoribus in reſultanti æquatione ſubſtitutis; & quantitates oriuntur deſideratæ.

Fig. 57. Ex. Gr. Sit quantitas data quæcunque functio linearum EF, FG, GH, &c. AB, BC, CD, &c. arcuum curvæ AB, BC, CD, &c. cujus inveniatur fluxio, quæ nihilo fiat æqualis, & facile ex æquatione reſultanti exterminentur fluxiones e diverſis quantitatibus EF, FG, &c. AB, BC, &c. arcubus AB, BC, &c. areis ABC, &c. & conſit exemplum.

Cor. 1. Si pro quibuſdam curvæ arcubus ſubſtituantur rectæ lineæ, quarum internus angulus haud major ſit quam duo recti, tum omnia præcedentia exempla ad hos caſus applicari poſſunt, e. g. Fig. *A*. Sit curva MAP, cujus axis ſit AP & ejus ordinata PM, & per ex. 1. ſi tangens ED biſecetur in duas æquales partes CE & DC in puncto contactûs C, tum triangulum DEP erit minimum, quod circumſcribit datam curvam; etiamque ducantur lineæ CF & CG parallelæ lineis DP & EP; & erit parallelogrammum CP maximum, quod in datâ ovali inſcribi poſſit.

Ex. 2. Sit $FP^{m-1} \times FE = FC^m$; & erit ſumma $FP^m + CF^m$ maxima: ſit arc. $CM^{m-1} \times CE = CF^m$, tum erit $CF^m +$ arc. CM^m maxima quantitas. Sit $m \times$ arc. $CM \times FE + o \times FP \times CE = n \times FP \times$ arc. CM; & erit $FP^m \times CF^n \times$ arc. CM^o maximum hujuſce generis contentum: ſit $FE \times CF = m \times PFCMP$ & erit contentum $CF \times \sqrt[m]{PFCMP}$ maximum.

His autem caſibus curva plura forſan maxima admittat, quorum plura vel omnia ſint æqualia, vel unum majus ~~erit~~ ſit quam reliqua.

Eadem

Eadem mutatis mutandis dici poſſunt de variis curvarum partibus, ſi modo ad axem continuo ſint concavæ vel convexæ, & ſic de rectis lineis, &c.

Fig. 57. Cor. 2. Si vero dentur quædam relationes inter prædictas lineas *E F*, *F G*, &c. *A B*, *B C*, &c. arcus, &c. & quædam functio ex his requiratur eſſe maxima vel minima. E datis relationibus & e datâ functione maximam vel minimam eſſe ſuppoſita inveniri poſſint æquationes quæſitæ, e. g. ſupponantur omnes lineæ vel circumſcripti vel inſcripti in datâ ovali polygoni æquales eſſe, & requiratur ut area vel ſumma laterum ſit maxima vel minima : tum utriuſque conditionis habenda eſt ratio ; ſed de his nimis.

3. Cum autem ad omne punctum referuntur lineæ, curvæ, &c. inter quas requiruntur æquationes relationem exprimentes.

Invenientur linearum, arearum, &c. ad ſingula puncta incrementa, & exinde ſæpe erui poſſint æquationes quæſitam relationem inter eas lineas, areas, &c. exprimentes ; e. g. ſi linea, area, &c. ad omne punctum poſſit eſſe maxima, tum earum incrementa nihilo fiunt æqualia ; ergo lineæ, areæ, &c. ſemper manent inter ſe æquales. Hujus caſus ſubſequens erit.

Fig. 58. Ex. 1. Circa ovalem nullum habentem contrariæ flexuræ punctum, deſcribantur duo polygona (n) laterum *E F G H*, &c. & *P Q R S*, &c. & ita ut utriuſque polygoni latera ſecentur in duas æquales partes in punctis contactuum (*A*, *B*, *C*, *D*, &c. & α, β, γ, δ, &c. reſpective), & ſi inter ea polygona ad omne curvæ punctum deſcribi poteſt polygonum, ita ut ſingula polygoni latera in duas æquales partes in punctis contactuum dividi poſſint, tum polygonum (*E F G H*, &c.) erit æquale polygono (*P Q R S*, &c.)

DEMONSTRATIO.

Hæc polygona per exemplum minima ſunt, quæ a reſpectivis punctis (*A* & α) incipiunt, & circa ovalem deſcribi poſſunt, deſcribantur etiam polygona incipientia a punctis inter *A* & α poſitis, & ita

ita ut ſint minima, quæ a reſpectivis punctis incipiunt. Tum, quoniam polygonum continuo eſt minimum omnium, quæ deſcribi poſſunt circa ovalem, nullum habet incrementum; & conſequenter inter ſe æqualia continuo manent polygona, & exinde ſequitur polygonum (*EFGH*, &c.) æquale eſſe polygono (*PQRS*, &c.)

Fig. 59. Cor. 1. Deſcribatur circa ovalem prædictam triangulum (*EFG*,) cujus latera (*EF*, *FG*, *EG*) reſpective dividantur in æquales partes ($AE = AF$, $BF = BG$, $CG = CE$) in punctis contactuum (*A*, *B*, *C*) i. e. ſit triangulum (*EFG*) minimum, quod circa ovalem deſcribi poteſt, & jungantur puncta contactuum (*A*, *B*, *C*); triangulum (*ABC*) erit maximum, vel maximo æquale, quod in datâ ovali inſcribi poſſit; facile enim conſtat lineam (*AB*) parallelam eſſe tangenti (*EG*), lineam (*BC*) parallelam tangenti (*EF*), & lineam (*AC*) tangenti *FG*; & conſequenter triangulum (*ABC*) maximum vel maximo æquale, quod in datâ ovali deſcribi poteſt. Et exinde e lemmate primo conſtat maxima triangula in ovali hujuſce generis inſcripta eſſe ad minima, quæ circumſcribi poſſint, in ratione 1 : 4.

Fig. 60. Cor. 2. Deſcribatur circa ovalem prædictam trapezium (*EFGH*), cujus latera in æquales partes per ſucceſſiva contactuum puncta (*A*, *B*, *C*, *D*) dividuntur; jungantur hæc contactuum puncta, trapezium (*ABCD*) exinde factum erit parallelogrammum, & ejus area dimidium areæ circumſcripti trapezii. Hoc coroll. ſequitur e lem. 2.

Eig. 61. Ex. 2. In ovali ad punctum ſemper cavâ inſcribantur duo (n laterum) polygona (*ABCDEF*, &c. & $\alpha\beta\gamma\delta\varepsilon$, &c.) ita ut diagonalis *AC* parallela ſit lineæ tangenti ovalem in puncto contactûs (*B*), & ſic diagonales *BD*, *CE*, *DF*, &c. parallelæ ſint reſpectivis lineis curvam tangentibus in punctis contactuum *C*, *D*, *E*, &c. & ſimiliter ſit diagonalis $\alpha\gamma$ parallela lineæ tangenti ovalem in puncto β; & lineæ $\beta\delta$, $\gamma\varepsilon$, &c. ſint parallelæ lineis tangentibus ovalem in punctis (γ, δ, &c.) ſi inter ea puncta ad ſingulum curvæ punctum deſcribi poſſit hujuſmodi polygonum; tum erunt duo polygona *ABCDE*, &c. $\alpha\beta\gamma\delta\varepsilon$, &c. inter ſe æqualia.

Hoc problema eâdem methodo, quâ præcedens demonſtrari poſſit.

Fig. 59. Cor. 1. Sit polygonum prædictum triangulum *(ABC)*; circumſcribatur triangulum *(EFG)*, cujus puncta contactûs ſunt *(A, B, C)*; triangulum *EFG* erit maximum, quod in data ovali inſcribi poſſit, & circumſcriptum erit ad inſcriptum :: 4 : 1.

Fig. 60. Cor. 2. Sit polygonum trapezium *(ABCD)*; circumſcriptum trapezium, cujus puncta contactuum ſint *(A, B, C, D)*, erit parallelogrammum; & habebit ad inſcriptum *(ABCD)* eandem rationem, quam habet numerus binarius ad unitatem.

Fig. 61. Ex. 3. In ovali ſibi ſemper concavâ inſcribantur duo *(n* laterum) polygona *(ABCDE*, &c. & $\alpha\beta\gamma\delta\varepsilon$, &c.) ita ut lineæ *AB*, *BC*; *CD*, *DE*; &c. æquales angulos cum lineis ovalem tangentibus in punctis contactuum *(B, C, D*, &c.) utrinque faciant; & ſic lineæ $\alpha\beta$, $\beta\gamma$; $\gamma\delta$, $\delta\varepsilon$; &c. æquales angulos cum lineis tangentibus ovalem in punctis contactuum β, γ, δ, &c. ſi inter puncta *A* & α perpetuo deſcribi poſſint polygona ejuſdem generis; tum ſummæ polygonorum laterum erunt reſpective inter ſe æquales, i. e. $AB + BC + CD + DE +$ &c. $= \alpha\beta + \beta\gamma + \gamma\delta + \delta\varepsilon +$ &c.

Fig. 60. Cor. Sit polygonum *ABCD* trapezium; ad puncta contactuum *A, B, C, D* ducantur tangentes *EF, FG, GH, HE*; circa trapezium *EFGH* deſcribi poteſt circulus, qui tranſit per puncta *E, F, G, H* reſpective.

Conſtat e lemmate 4. 2. hujus capitis.

Eig. 58. Ex. 4. Sint duo polygona *(EFGH*, &c. *PQRS*, &c.*)* *n* laterum circa ovalem prædictam deſcripta; & ſint puncta contactuum illius polygoni *(EFGH*, &c.*)* reſpective *A, B, C, D*, &c. hujus (*PQRS*, &c.) vero α, β, γ, δ, &c. & $FA : AE ::$ $\overline{\text{tang.} + \text{ſec.}}$ comp. $< AEp$: $\overline{\text{tang.} + \text{ſec.}}$ comp. $< AFG$; $FB : BG ::$ $\overline{\text{tang.} + \text{ſec.}}$ comp. $< BGC$: $\overline{\text{tang.} + \text{ſec.}}$ comp. $< BFA$; & ſic deinceps: & eodem modo ſecentur polygoni *PQRST*, &c. latera in punctis contactuum α, β, γ, δ, &c.

&

& ſi inter puncta A & α continuo deſcribi poſſint polygona, quorum latera in prædictâ ratione ſecantur; tum erit ſumma laterum $EF + FG + GH +$ &c. $= PQ + QR + RS +$ &c.

In genere inſcribantur in datâ ovali vel ſolido vel circumſcribantur circa ovalem vel ſolidum lineæ, areæ, vel ſolida, ita ut lineæ, areæ vel ſolida, vel eorum quælibet functiones ſint maxima vel minima quæ inſcribi vel circumſcribi poſſunt, & mutetur punctum contactûs, & continuo ſint lineæ, areæ, ſolida, vel eorum functiones, maxima; tum lineæ, areæ, ſolida vel eorum functiones erunt inter ſe æqualia.

Ex hypotheſi enim lineæ, areæ, ſolida, vel eorum functiones ſunt maxima, & conſequenter eorum incrementa nihilo ſunt æqualia, ergo lineæ, areæ, ſolida vel eorum functiones continuo manent inter ſe æqualia.

Et ſic deduci poſſint theoremata, quæ non modo includant quantitates, quæ ſemper ſunt maximæ; ſed eas etiam, quæ datam inter ſe habeant relationem.

Fig. 56. Si data functio linearum AB, BC, CD, &c. ſit algebraica, tum incrementum datæ functionis ad puncta contactuum A, B, C, D, &c. idem erit ac in rectilineo polygono $EFGH$, &c. cujus puncta contactuum ſint A, B, C, D, &c. ſi vero data functio ſit functio linearum AB, BC, &c. & earum primarum fluxionum, tum idem erit incrementum functionis ac in curvâ vel curvis, quarum radii curvaturæ & tangentes ad puncta A, B, C, D, &c. eædem ſunt; & ſic deinceps.

Sint duæ curvæ, quarum dantur æquationes deſignantes relationes inter earum abſciſſas & correſpondentes ordinatas: inſcribatur una curva in alterâ; & numerus punctorum, in quibus una curva tangere poteſt alteram, plerumque pendet e numero incognitarum quantitatum in datis æquationibus contentarum. Et ſic de duobus ſolidis, quorum unum inſcribitur in altero: ſi vero requiratur, ut curva inſcripta vel circumſcripta, vel ſolidum, &c. ſit maximum, vel minimum, e principiis prius traditis & vulgari algebrâ petenda eſt ſolutio.

4.

4. Invenire relationes inter lineas, areas & ſolida e diverſis curvis derivata.

Datis æquationibus relationes inter quaſdam datarum curvarum lineas, areas, &c. exprimentibus, inveniatur relatio inter has lineas, areas, &c. & lineas, areas, &c. inter quas requiruntur relationes, quibus æquationibus reductis reſultant æquationes relationem quæſitam exprimentes.

Quærenti facile occurrent exempla.

5. Invenire curvas, quæ datâ proprietate vel datis proprietatibus gaudent.

Proprietas data vel reſpicit unum punctum indeterminatum vel plura: ſi ~~vero~~ unum ſolummodo reſpiciat punctum, tum ſemper e datâ proprietate erui poſſit curva; ſi vero plura n admittat puncta indeterminata, tum ex aſſumptis $n-1$ punctis deduci poſſit reliquum, e. g. ſi data proprietas curvæ hujuſmodi ſit, ut ſumma linearum e duobus datis punctis ad punctum indeterminatum ductarum ~~continuo~~ ſit maxima; tum facile deduci poſſit curva quæſita, & invenietur conica ellipſis. Fig. 56. Si vero ſit proprietas datæ curvæ, ut ad omne punctum datæ curvæ ita duci poſſit triangulum vel polygonum ($EFGH$, &c.) n laterum curvam circumſcribens, ut omnia latera (EF, FG, GH, &c.) ad puncta contactuum (A, B, C, &c.) biſecentur: hæc vero proprietas reſpicit plura indeterminata puncta (A, B, C, D, &c.) aſſumantur igitur omnia puncta (A, B, C, D, &c.) præter unum tanquam determinata, & ducantur lineæ EF, FG, GH, &c. IE; ita ut $EA=AF$, $FB=BG$, $GC=CH$, $HD=DI$, ducatur linea IE, & biſecetur linea IE in duas æquales partes $ID'=D'E$, & D' erit punctum in curvâ quæſita.

Si vero plures (m) dentur proprietates independentes, & ſint n diverſa puncta indeterminata, tum aſſumi poſſint ad libitum $n-m-1$ puncta, i.e. ſi dentur m independentes æquationes n incognitas lineas, &c. involventes, tum ſcribi poſſint pro $n-m-1$ incognitis lineis, &c. ad libitum quæcunque aliæ lineæ, &c.

Pro-

Proprietates quædam novæ ſectionum conicarum.

THEOR. XVI.

Fig. *A*. Sit ellipſis $APBQCRDSET$, &c. deſcribatur circa eam duo polygona ($abcdef$, &c. $pqrstv$, &c.) eundem laterum numerum habentia, & quorum latera ad reſpectiva contactuum puncta ($APBQCRDS$, &c.) in duas æquales partes dividuntur, i. e. $aA = Ab$, $bB = Bc$, $cC = Cd$, &c. $pP = Pq$, $qQ = Qr$, $rR = Rs$, &c. & erit ſumma quadratorum ex ſingulis unius polygoni lateribus æqualis ſummæ quadratorum ex ſingulis alterius polygni lateribus; i. e. $ab^2 + bc^2 + cd^2 + de^2 + ef^2 +$ &c. $= pq^2 + qr^2 + rs^2 + st^2 + tv^2 +$ &c.

Cor. . Ducantur lineæ AB, BC, CD, DE, EF, &c. PQ, QR, RS, ST, TV, &c. & erit $AB^2 + BC^2 + CD^2 + DE^2 +$ &c. $= PQ^2 + QR^2 + RS^2 + ST^2 +$ &c.

THEOR. XVII.

Iiſdem poſitis, ſit O centrum ellipſeos, & ducantur lineæ OA, OP, OB, OQ, OC, OR, OD, OS, &c. & erit $OA^2 + OB^2 + OC^2 + OD^2 +$ &c. $= OP^2 + OQ^2 + OR^2 + OS^2 +$ &c.

Cor. Ducantur etiam lineæ Oa, Op, Ob, Oq, Oc, Or, Od, Os, &c. & erit $Oa^2 + Ob^2 + Oc^2 +$ &c. $= Op^2 + Oq^2 + Or^2 + Os^2$ $+$ &c.

Hæc etiam vera ſunt de polygonis inter conjugatas hyperbolas eodem modo deſcriptis.

THEOR. XVIII.

Fig. *B*. Sit conica ſectio $MPQRSTM'$, &c. cujus diameter ſit AL, & ejus ordinata ML; ſit $Mp = Mv$, & conſequenter $Lp = Lv$.

Ducantur lineæ pq, qr, rs, st, tv, &c. quæ reſpective tangant conicam ſectionem in punctis P, Q, R, S, T, &c. & erit contentum $pP \times qQ \times rR \times sS \times$ &c. $= Pq \times Qr \times Rs \times St \times Tv \times$ &c. vel quod

quod idem eſt, ſumma omnium hujus generis rationum (*Pp*: *Pq*, *Qq* : *Qr*, *Rr* : *Rs*, *Ss* : *St*, &c.) erit nihilo æqualis.

Fig. *C*. Cor. 1. Sit ellipſis *PQRSTV*, &c. circa eam deſcribatur quodcunque polygonum (*pqrstuw*, &c.) cujus latera reſpective tangant ellipſim in punctis *P*, *Q*, *R*, *S*, *T*, *V*, &c. & erit contentum *Pp* × *Qq* × *rR* × *sS* × × *tT vV* × &c. = *Pq* × *Qr* × *Rs* × *St* × *Tv* × *Vw* × &c.

Cor. 2. Ducantur lineæ *PQ*, *QR*, *RS*, *ST*, &c. & pro ſinubus angulorum *WPp*, *QPq*, *RQr*, *QRr*, *SRs*, *TSt*, &c. ſcribantur reſpective *a*, *p*, *b*, *q*, *c*, *r*, *d*, *s*, &c. & erit *abcd*, &c. = *pqrs*, &c.

Et ſic de polygonis circa hyperbolas deſcriptis.

THEOR. XIX.

Fig. *D*. Sit ellipſis *PAQBRCSDTEVF*, &c. circa eam deſcribantur duo polygona *abcdef*, &c. *pqrstu*, &c. eundem laterum numerum habentia; eorum latera *ab*, *bc*, *cd*, *de*, *ef*, &c. *pq*, *qr*, *rs*, *st*, *tv*, &c. reſpective tangant ellipſim in punctis *A*, *B*, *C*, *D*, *E*, *F*, &c. *P*, *Q*, *R*, *S*, *T*, *V*, &c. & ſit *aA* : *Ab* :: *pP* : *Pq*, & *bB* : *Bc* :: *qQ* : *Qr*, & *cC* : *Cd* :: *rR* : *Rs*, & *dD* : *De* :: *sS* : *St*, & ſic deinceps. Et area polygoni *abcdef*, &c. æqualis erit areæ polygoni *pqrstv*, &c.

Fig. *E*. Cor. 1. Duo parallelogramma (*abcd* & *pqrs*) circa datæ ellipſeos conjugatas diametros (*AC* & *BD*, *PR* & *QS*) deſcripta, erunt inter ſe æqualia.

In hoc caſu enim *aA* = *Ab*, *bB* = *Bc*, *cC* = *Cd*, *dD* = *Da*; & *pP* = *Pq*, *qQ* = *Qr*, *rR* = *Rs*, *sS* = *Sp*; & conſequenter *aA* : *Ab* :: *pP* : *Pq*, & *bB* : *Bc* :: *qQ* : *Qr*, & ſic deinceps, ergo per theorema hæc duo parallelogramma erunt inter ſe æqualia, quæ eſt notiſſima ellipſeos proprietas.

Fig. *D*. Cor. 2. Ducantur lineæ *AB*, *BC*, *CD*, &c. & *PQ*, *QR*, *RS*, &c. & erit polygonum inſcriptum *ABCD*, &c. æquale polygono *PQRS*, &c.

Cor. 3. Sint *aA* = *Ab*, *bB* = *Bc*, *cC* = *Cd*, *dD* = *De*, &c.

&

& $pP = Pq$, $qQ = Qr$, $rR = Rs$, $sS = St$, &c. tum ea polygona erunt minima, quæ circa datam ellipſim deſcribi poſſint, & quæ eundem habent numerum laterum, & quorum puncta contactuum ſint reſpective *A* & *P*: & conſtat hæc polygona eſſe inter ſe æqualia.

Cor. 4. Sint $aA = Ab$, $bB = Bc$, $cC = Cd$, &c. ſicut in præcedente corollario: ducantur lineæ *AB*, *BC*, *CD*, *DE*, &c. & polygonum *A B C D E*, &c. (*n* laterum) majus erit quam quodvis aliud inſcriptum polygonum (*n*) laterum, cujus punctum contactûs ſit *A*: ducantur etiam lineæ *AC*, *BD*, *C E*, *DF*, &c. & erit linea *AC* parallela lineæ tangenti ellipſim in puncto *B*, linea *BD* parallela lineæ tangenti in puncto *C*, linea *CE* parallela lineæ tangenti in puncto *D*, & ſic deinceps.

Cor. 5. Sint *ABCD*, &c. *PQRS*, &c. duo polygona *n* laterum; & ſint maxima, quæ deſcribi poſſint e reſpectivis punctis *A* & *P* incipientia; tum polygonum *A B C D*, &c. erit æquale polygono *P Q R S*, &c.

Cor. 6. Minimum polygonum *n* laterum, quod circa ellipſim deſcribi poſſit, erit ad maximum polygonum, quod in prædictâ ellipſi inſcribi poſſit :: 1 : coſ. arc. $\frac{360^\circ}{n}$ $\overline{}^{2}$. Et ſic de polygonis inter conjugatas hyperbolas deſcriptis.

THEOR. XX.

Fig. *F*. In datâ ellipſi inſcribantur duo (*n*) laterum polygona *a b c d e*, &c. & *p q r s t*, &c. ad puncta reſpectiva *a*, *b*, *c*, *d*, *e*, &c. *p*, *q*, *r*, *s*, *t*, &c. ducantur tangentes *AB*, *BC*, *C D*, *D E*, &c. & *PQ*, *QR*, *RS*, *ST*, &c. & ſint $\angle abB = \angle cbC$, $\angle bcC = \angle dcD$, $\angle cdD = \angle edE$, &c. & $\angle pqQ = \angle rqR$, $\angle qrR = \angle srS$, $\angle rsS = \angle tsT$, & ſic deinceps. Et erit ſumma laterum $ab + bc + cd + de +$ &c. $= pq + qr + rs + st +$ &c.

Fig. *G*. Cor. Ducatur in ellipſi polygonum *a b c d e*, &c. (*n*) laterum methodo ſupra traditâ; inſcribatur etiam aliud polygonum

a b k l m, &c. (*n*) laterum quovis alio modo, cujus unus angulus ponitur ad punctum (*a*), & ſumma $ab + bc + cd + de +$ &c. major eſt quam ſumma $ab + bk + kl + lm +$ &c.

THEOR. XXI.

Fig. *F*. Deſcribantur circa datam ellipſim duo (*n*) laterum polygona *A B C D E*, &c. & *P Q R S T*, &c. quorum puncta contactuum reſpective ſunt *a*, *b*, *c*, *d*, *e*, &c. & *p*, *q*, *r*, *s*, *t*, &c. Et ſint $\overline{\text{tang.} + \text{ſec.}}$ comp. $\angle aBb : \overline{\text{tang.} + \text{ſec.}}$ comp. $\angle cCb :: bC : bB$; & $\overline{\text{tang.} + \text{ſec.}}$ comp. $cCb : \overline{\text{tang.} + \text{ſec.}}$ comp. $\angle cDd :: cD : cC$; & $\overline{\text{tang.} + \text{ſec.}}$ comp. $\angle cDd : \overline{\text{tang.} + \text{ſec.}}$ comp. $\angle eEd : Ed : dD$, &c. Et ſic $\overline{\text{tang.} + \text{ſec.}}$ comp. $< pQq : \overline{\text{tang.} + \text{ſec.}}$ comp. $\angle qRr :: qR : qQ$, & $\overline{\text{tang.} + \text{ſec.}}$ comp. $\angle qRr : \overline{\text{tang.} + \text{ſec.}}$ comp. $\angle sSr :: Sr : rR$, & $\overline{\text{tang.} + \text{ſec.}}$ comp. $\angle sSr : \overline{\text{tang.} + \text{ſec.}}$ comp. $\angle tTs :: Ts : sS$; & ſic deinceps.

Et erit ſumma laterum $AB + BC + CD + DE +$ &c. $= PQ + QR + RS + ST +$ &c.

Fig. *H*. Cor. Deſcribatur circa ellipſim polygonum (*n*) laterum *A B C D E*, &c. methodo, quæ prius data fuit; deſcribatur etiam circa ellipſim aliud polygonum *G H K L M*, &c. (*n*) laterum quâvis aliâ methodo, cujus unum punctum contactûs (*a*) eſt punctum contactûs polygoni *A B C D E*, &c.

Et ſumma $AB + BC + CD + DE +$ &c. minor erit quam ſumma $GH + HK + KL + LM +$ &c.

Conſimiles proprietates affirmari poſſunt de polygonis circa hyperbolas deſcriptis.

THEOR. XXII.

Fig. *I*. Rotetur conica ſectio circa diametrum ejus (*AL*), & ſit *M A M'*, &c. ſolidum exinde generatum; ſint *p q*, *q r*, *r s*, *s t*, *t v*, *v w*, *w p*, &c. lineæ, quæ tangant ſolidum in reſpectivis punctis *P*, *Q*, *R*, *S*,

S, T, V, W, &c. & erit contentum $pP \times qQ \times rR \times sS \times tT \times vV \times wW \times$ &c. $= Pq \times Qr \times Rs \times St \times Tv \times Vw \times$ &c.

THEOR. XXIII.

Fig. *K*. Sit ellipſis *APBQCR*, &c. rotetur circa ~~diametrum~~ axem ejus *BD*; & circa conjugatas diametros (*AC* & *BD*, *PR* & *QS*) deſcribantur elliptici cylindri (*pqrs* & *abcd*) ſolidum generatum circumſcribentes; & erunt hi duo cylindri inter ſe æquales.

Cor. Et ſic de quibuſcunque duobus ſolidis circa prædicta generata ſolida deſcriptis, quorum latera (rectilinea) continuo eâdem ratione ad puncta contactuum dividuntur: erunt enim hæc duo ſolida inter ſe æqualia.

Et ſimiliter ſolida in prædictis generatis ſolidis e punctis contactuum prædictorum circumſcriptorum ſolidorum deſcripta, erunt inter ſe æqualia.

Eadem etiam affirmari poſſint de ſolidis inter ſolida e rotatione conjugatarum hyperbolarum circa axes ſuos, &c. generata ſimiliter inſcriptis.

THEOR. XXIV.

Fig. *L*. Si in prædictâ ſphæroide inſcribantur coni *ABC* & *DEF*, qui ſint maximi eorum, qui eaſdem habent vertices (*A* & *D*) reſpective; erunt hi duo coni inter ſe æquales: & ſi duo etiam inſcribantur cylindri (*ABDE* & *GHIK*), qui maximi ſint eorum, qui unum punctum contactûs *A* & *G* idem reſpective habent; tum hi duo cylindri erunt inter ſe æquales.

Si etiam deſcribantur circa ſphæroidem duo coni *EFG* & *ABC*, qui ſint minimi eorum, qui unum punctum contactûs (*K* & *L*) idem reſpective habent, tum hi duo coni circumſcripti erunt inter ſe æquales. Et ſic circumſcribantur duo cylindri *ABCD* & *EFGH*, qui ſint minimi circumſcriptorum, qui unum punctum contactûs (*K* & *L*) idem reſpective habent, tum hi duo circumſcripti cylindri erunt inter ſe æquales.

 Per

Per cylindros & conos haud eos ſolum intelligo, qui habent circulos pro ſuis baſibus, ſed etiam ellipſes.

Eadem etiam vera ſunt de inſcriptis & circumſcriptis polygonis, quæ maxima vel minima ſunt; omnia hæc theoremata etiam mutatis mutandis de talibus figuris vel circa conicas hyperbolas deſcriptis, vel in iiſdem inſcriptis, affirmari poſſunt.

Et conſimiles propoſitiones de ſuperficiebus figurarum inſcriptarum & circumſcriptarum deduci poſſint.

THEOR. XXV.

Omnia parallelogramma circa circulum deſcripta ſunt rhombi, & omnia ~~cujuſcunque~~ parallelogrammi duæ diametri ſeſe in circuli centro & ad rectos angulos interſecant, & conſequenter duæ diagonales parallelogrammi circa ellipſim deſcripti erunt inter ſe conjugatæ diametri.

PROB. XXIX.

Fig. 29. Inſcribere in datâ ellipſi (*F F G H*) parallelogrammum dato parallelogrammo (*A B C D*) ſimile.

Inveniantur datæ ellipſeos duæ conjugatæ diametri (*E F*, *G H*), quæ angulum inter ſe æqualem parallelogrammi angulo (*A B C*) faciunt. Sit *C* centrum & in lineâ (*E F*) capiatur $CP = AB$; per punctum *P* ducatur linea $PQ = BC$ & parallela diametro *HG*; jungatur linea *CQ* ſecans curvam in *s*, & per *s* ducatur linea *st* parallela diametro *HG*; & erit parallelogrammi quæſiti latus; ducantur lineæ *sm* & *tn* parallelæ diametro *EF*, & erunt duo reliqua parallelogrammi latera.

Cor. 1. Si data ellipſis duas non habet conjugatas diametros angulum inter ſe dato parallelogrammi angulo (*ABC*) æqualem facientes, tum haud recipiet ſolutionem problema.

Cor. 2. Hinc conſtat methodus ducendi duas lineas per centrum datæ conicæ ſectionis datam inter ſe habentes relationem, & datum angulum ſecum facientes.

PROB.

PROB. XXX.

Fig. 30. Inſcribere in datâ ellipſi ($EFGH$) trapezium, quod erit ſimile dato trapezio ($ABCD$).

Ducantur duæ dati trapezii diagonales AC & BD, ſeſe ſecantes in puncto (n), & ſint nA major quam Cn, & Bn quam nD. Per præcedens problema ducantur duæ lineæ (LM & RS) per ellipſeos centrum C, angulum (LCR) inter ſe habentes angulo (AnB), quem inter ſe faciunt diagonales AC & BD, æqualem; & ita ut $LM^2 : RS^2 :: An \times nC : Bn \times nD$.

Ducantur NP & TV conjugatæ diametrorum LM & RS; ſint etiam $2CP : Pp :: AC : nC$, & $2CT : Tt :: BD : nD$; & ſuper CL & Cp tanquam ſemiconjugatas diametros deſcribatur ellipſis $LpbPL$, & ſuper ſemiconjugatas diametros (RC & Ct) deſcribatur ellipſis $RtbSR$; & interſecent hæ duæ ellipſes ſeſe in puncto b, per b ducantur duæ lineæ hab & hcd reſpective parallelæ lineis LM & RS, & erunt quæſiti parallelogrammi diagonales.

Angulus enim, quem inter ſe faciunt lineæ hab & hcd, æqualis eſt angulo (AnB), & diagonalium ſegmenta ab, hb, cb, bd erunt inter ſe reſpective in ratione linearum An, nC, Bn, nD.

PROB. XXXI.

Fig. 31. Circa datam ellipſim ($EFGH$) parallelogrammum deſcribere, quod parallelogrammo ($ABCD$) erit ſimile.

Ducantur duæ dati parallelogrammi diametri (AC & BD) ſeſe interſecantes in centro (N), & ad datum angulum ANB. Ellipſeos inveniantur duæ conjugatæ diametri (EF & GH), quæ faciunt angulum inter ſe æqualem dato angulo ANB; & hæ conjugatæ diametri (EF, GH) juxta theor. 24. erunt diametri parallelogrammi quæſiti.

Scribantur pro ſemiconjugatis datæ ellipſeos diametris (EF, GH) reſpective t, c; & pro dati parallelogrammi ſemidiametris (AN, NB) reſpective

reſpective r & s; aſſumatur abſciſſa $CK = \sqrt[2]{\frac{t^4 s^2}{t^2 s^2 + r^2 c^2}}$; per K ducatur ejus ordinata KL; per ellipſeos punctum L ducatur linea MN curvam tangens, & diametros EF & GH in punctis M & N ſecans; & MN erit latus parallelogrammi quæſiti, quo dato facile etiam deduci poſſint reliqua latera.

Cor. Si nullæ ſint datæ ellipſeos conjugatæ diametri, quæ angulum inter ſe faciunt æqualem angulo, quem inter ſe habent dati parallelogrammi diametri, tum haud circumſcribi poteſt parallelogrammum dato parallelogrammo ſimile.

THEOR. XXVI.

Fig. 25. Sit circulus $ABDEFG$, &c. P, cujus centrum eſt C & radius unitas; ſint etiam arcus AB, BD, DE, EF, &c. inter ſe æquales, pro chordâ circuli (PB) ſcribatur p, & aſſumatur quadratica æquatio $x^2 - px + 1 = 0$; ſupponantur duæ hujuſce æquationis radices α & β, tum erunt reſpectivæ chordæ $PB = \alpha + \beta$, $PD = \alpha^2 + \beta^2$, $PE = \alpha^3 + \beta^3$, $PF = \alpha^4 + \beta^4$, & ſic deinceps.

DEMONSTRATIO.

Sit $BP = p = a$, tum e vulgari trigonometriâ ſequitur $PD = pa - 2 = b$, $PE = pb - a = c$, $PF = pc - b = d$, & ſic deinceps. Sit quadratica æquatio $x^2 - px + 1 = 0$, tum per vulgarem algebram cognitum eſt $\alpha + \beta = p = a = BP$, $\alpha^2 + \beta^2 = pa - 2 = PD$, $\alpha^3 + \beta^3 = pb - a = PE$, $\alpha^4 + \beta^4 = pc - b = PF$, & ſic deinceps.

Cor. 1. Sit arcus $AG = n \times AB$, & chorda $BP = p$, ſit etiam quadratica æquatio ($x^2 - px + 1 = 0$,) cujus radices ſint α & β; tum chorda ($PG = \alpha^n + \beta^n$) erit per prob. prim. noſtr. medit. algebr. $p^n - np^{n-2} + n \times \frac{n-3}{2} p^{n-4} - n \times \frac{n-4}{2} \times \frac{n-5}{3} p^{n-6} + n \times \frac{n-5}{2} \times \frac{n-6}{3} \times \frac{n-7}{4} p^{n-8} - \text{\&c.} = PG$.

Cor.

Cor. 2. Duæ radices α & β quadraticæ æquationis $x^2 - px + 1 = 0$, ſunt reſpective $\sqrt{\frac{1}{4}p^2 - 1} + \frac{1}{2}p$ & $\frac{1}{2}p - \sqrt{\frac{1}{4}p^2 - 1}$, & conſequenter $\overline{\frac{1}{2}p + \sqrt{\frac{1}{4}p^2 - 1}}^n + \overline{\frac{1}{2}p - \sqrt{\frac{1}{4}p^2 - 1}}^n = \alpha^n + \beta^n = p^n - np^{n-2} + n \times \frac{n-3}{2} p^{n-4} - n \times \frac{n-4}{2} \times \frac{n-5}{3} p^{n-6} +$ &c.

Cor. 3. Hinc reduci poteſt in quadraticos ejus diviſores quantitas $(\pi^{2n} - \alpha\pi^n + 1)$. Sit quadratica æquatio $x^2 - \frac{\pi^2 + 1}{\pi} x + 1 = 0$; cujus radices erunt reſpective π & $\frac{1}{\pi}$; & aſſumantur duo arcus AB & AG, quorum AG ſit $n \times AB$; ſit etiam chorda $PG = \alpha$, $PB = \frac{\pi^2 + 1}{\pi}$, & radius unitas; tum erit per theorema $PG = \alpha = \pi^n + \frac{1}{\pi^n}$; & conſequenter $\pi^{2n} - \alpha\pi^n + 1 = 0$, datâ chordâ (α), invenitur arcus PG, & ejus ſupplementum AG; & exinde arcus $AB = \frac{AG}{n}$, vel $\frac{2 \times 360° + AG}{n}$, vel $\frac{4 \times 360° + AG}{n}$, vel $\frac{6 \times 360° + AG}{n}$, vel $\frac{8 \times 360° + AG}{n}$, &c. dato igitur arcu AB inveniatur ejus ſupplementi chorda PB, pro quâ ſcribatur β, & erit $\frac{\pi^2 + 1}{\pi} = \beta$, quâ æquatione reductâ reſultat quadraticus diviſor $(\pi^2 - \beta\pi + 1)$ quæſitus.

Eodem fere modo inveniri poſſint diviſores quantitatum $\pi^n + 1$ vel $\pi^n - 1$; inveniantur enim per methodum præcedentem diviſores quantitatum $\pi^{2n} + 2\pi^n + 1$ vel $\pi^{2n} - 2\pi^n + 1$; quibus cognitis conſtant diviſores quæſiti.

Quantitas etiam $\pi^{2n} - \alpha\pi^n \pm 1$, ubi α major ſit quam 2, facile transformari poteſt in quantitates præcedentium formularum.

Hinc etiam inveniri poſſint diviſores quantitatum $\pi^{3n} - \alpha\pi^{2n} + \beta\pi^n - 1$ vel $\pi^{4n} - \alpha\pi^{3n} + \beta\pi^{2n} - \gamma\pi^n + 1$: facile enim transformari poſſint hæ æquationes per methodos in vulgari algebrâ notas in quantitates præcedentium formularum.

Et

Et ſic transformari poſſint plures æquationes in æquationes præcedentium formularum.

Cor. 4. Sit r radius, & a cujuſlibet arcûs $(180° - A)$ chorda, & chordæ ſupplementorum arcuum $\frac{A}{n}$, $\frac{2 \times 360° + A}{n}$, $\frac{4 \times 360° + A}{n}$, $\frac{6 \times 360° + A}{n}$, &c. erunt radices æquationis $x^n - n r^2 x^{n-2} + n \times \frac{n-3}{2} r^4 x^{n-4} - \&c. = a r^{n-1}$.

Fig. 25. Cor. 5. 1. Sit n impar numerus, & dividatur circuli peripheria in n æquales partes (AB, BD, DE, EF, FG, &c.) aſſumatur quodcunque punctum P in peripheriâ circuli, & ducantur chordæ PA, PB, PD, PE, &c. & erunt hæ chordæ (ſi modo negative & affirmative alternatim aſſumantur) radices æquationis $x^n - n r^2 x^{n-2} + n \times \frac{n-3}{2} r^4 x^{n-4} - n \times \frac{n-4}{2} \times \frac{n-5}{3} r^6 x^{n-6} + \&c. = a r^{n-1}$, ubi a ſit chorda arcûs $n \times PA - \overline{n-1} \times 180°$.

2. Sit n par numerus, & dividatur circuli peripheria in $\frac{n}{2}$ æquales partes (AB, BD, DE, &c.) aſſumatur quodcunque punctum P in peripheriâ circuli, & ducantur chordæ PA, PB, PD, PE, &c. & erunt hæ chordæ, & affirmative & negative aſſumptæ, radices æquationis $x^n - n r^2 x^{n-2} + n \times \frac{n-3}{2} r^4 x^{n-4} - \&c. = a r^{n-1}$, ubi a ſit chorda arcûs $n \times PA - \overline{n-1} \times 180°$.

Cor. 6. Sit m impar numerus, & minor quam n, & ſumma m poteſtatum e ſingulis his chordis erit nihilo æqualis.

Cor. 7. Sit m minor quam n, & ſumma $2m$ poteſtatum e ſingulis chordis, erit $\frac{1 \cdot 3 \cdot 5 \cdot 7 \ldots 2m-1}{1 . 2 . 3 \ldots m} \times 2^m \times n \times r^{2m}$. Facile conſtant hæc duo corollaria e problemate primo noſtrarum medit^m. algebr.

Cor. 8. Quoniam quadratum chordæ per diametrum $(2r)$ diviſum æquale

æquale eſt ejus verſo ſinui: ſumma m poteſtatis e ſingulis verſis ſinubus prædictarum chordarum erit $\frac{1 \cdot 3 \cdot 5 \cdot 7 \ldots 2m - 1}{1 \cdot 2 \cdot 3 \ldots m} \times n r^m$.

Fig. *a*. Sit curva ellipſis $ABDE$, &c. cujus æquatio ad abſciſſam axi pertinentem, & ejus correſpondentes ordinatas ſit $y^2 = p \times \overline{a^2 - x^2}$; dividatur ellipſis in (n) æquales areas ACB, BCD, DCE, &c. & ducantur lineæ AB, AD, AE, &c. e ſubſtitutione facile deduci poſſit ſumma quadratorum, &c. e ſingulis chordis AB, AD, AE, &c. ſed de his nimis.

PROB. XXXII.

Fig. 26. & θ.1. Sit n impar numerus, & dividatur peripheria circuli in n æquales partes $(AB, BD, DE, EF, FG, GH,$ &c.$)$ & a quolibet puncto P, quod haud ponitur in circuli peripheriâ, ducantur lineæ PA, PB, PD, PE, &c. invenire æquationem, cujus radices ſint PA, PB, PD, PE, &c.

A centro C ducatur linea CP, & producatur uſque donec ſecat peripheriam in puncto Q; ducantur lineæ QA, QB, QD, QE, &c. pro lineis PA, PB, PD, PE, &c. ſcribatur reſpective (z); & pro lineis QA, QB, QD, QE, &c. reſpective (x); pro radio unitas; & pro lineâ $CP\,(a)$: tum ſequitur $x^2 = \frac{z^2 + 2a - a^2 - 1}{a}$; quâ quantitate pro ſuo valore in æquatione $x^n - nx^{n-2} + n.\frac{n-3}{2}x^{n-4} - n.\frac{n-4}{2}.\frac{n-5}{3}x^{n-6} +$ &c. $= A$ (ubi A ſit chorda arcûs $n \times QA - \overline{n-1} \times 180°$,) ſubſtitutâ; reſultat æquatio $z^{2n} - n \times \overline{a^2 + 1}\, z^{2n-2} + n.\overline{\frac{n-1}{2}\,a^4 + 1 + n.\overline{n-2}\,a^2}\, z^{2n-4} -$ &c. $- \overline{a^{2n} - 2a^n + 1} - A^2 \times a^n = 0$, cujus ultimus terminus eſt $a^{2n} + \overline{A^2 - 2} \times a^n + 1$; hic vero ultimus terminus æqualis eſt contento $PA^2 \times PB^2 \times PD^2 \times PE^2$ &c.

P

Fig.

Fig. θ. Cor. Sit chorda $A = o$; erit $a^{2n} - 2a^n + 1 = PA^2 \times PB^2 \times PD^2 \times PE^2 \times$ &c. unde $a^n - 1 = PA \times PB \times PD \times PE \times$ &c. ſit $A = 2$, diameter circuli; erit $a^{2n} + 2a^n + 1 = PA^2 \times PB^2 \times PD^2 \times PE^2 \times$ &c. unde $a^n + 1 = PA \times PB \times PD \times PE$: & conſequenter; ſi circuli circumferentia, cujus centrum ſit C, dividatur in totidem æquales partes Ap, pB, Bq, qD, Dr, rE, sE, sF, &c. quot ſint unitates in $2n$; & ab omnibus diviſionum punctis ad punctum quodvis P in radio CA, ſi opus ſit, producto ſitum, ducantur rectæ PA, Pp, PB, Pq, PD, Pr, &c. deinde poſito $CA = 1$, $CP = a$; contentum ſub omnibus lineis PA, PB, PD, &c. ſumptis a diviſionum punctis per integrum circuitum, adæquabit $1 - a^n$ vel $a^n - 1$, prout P fuerit intra vel extra circulum; & contentum ſub reliquis $Pp \times Pq \times Pr \times Ps \times$ &c. in locis reliquis alternis adæquabit $1 + a^n$.

Idem etiam verum eſt, ſi n ſit par numerus.

Fig. 26. 2. Eodem modo, quo prius, dividatur peripheria circuli in $\frac{n}{2}$ æquales partes, & ducantur lineæ PA, PB, PD, PE, &c. QA, QB, QD, QE, &c. invenire æquationem, cujus radices ſint PA, PB, PD, &c. Pro lineis PA, PB, &c. ſcribatur (z); pro lineis QA, QB, &c. ſcribatur (x); pro radio unitas; & pro lineâ (CP) a; erit $x^2 = \frac{z^2 + 2a - a^2 - 1}{a}$; quâ quantitate pro ſuo valore in æquatione $x^n - nx^{n-2} +$ &c. $= A$ (ubi A ſit chorda arcûs $n \times QA - \overline{n-1} \times 180^\circ$) ſubſtitutâ; reſultat æquatio $z^n - \frac{n}{2} \times \overline{a^2 + 1}\, z^{n-2} + n \times \frac{n-2}{8}\, \overline{a^4 + 1} + n \times \frac{n-4}{4}\, a^2 z^{n-4} -$ &c. $\pm \overline{a^n + 1} - Aa^{\frac{n}{2}} = o$, cujus ultimus terminus $\pm \overline{a^n + 1} - Aa^{\frac{n}{2}}$ æqualis eſt contento $PA \times PB \times PD \times PE \times$ &c. $\times -PA \times -PB \times -$ &c. Erit $+\overline{a^n + 1}$ ſi $\frac{n}{2}$ ſit par numerus; ſin aliter $-\overline{a^n + 1}$.

Cor.

Cor. Dividatur circuli peripheria in n æquales partes AB, BD, DE, EF, &c. & ſit A vel $= 2$ vel $= -2$; ultimus terminus erit vel $a^n - 2a^{\frac{n}{2}} + 1 = PA^2 \times PD^2 \times PF^2 \times$ &c. vel $a^n + 2a^{\frac{n}{2}} + 1 = PB^2 \times PE^2 \times PG^2 \times$ &c. & exinde $a^{\frac{n}{2}} - 1 = PA \times PD \times PF \times$ &c. & $a^{\frac{n}{2}} + 1 = PB \times PE \times$ &c. ſit $A = o$, & ſit ultimus terminus $= a^n + 1$, in hoc caſu erit arcus $QA = \overline{n-1} \times \frac{180^\circ}{n}$: & ſic dividatur, fig. θ. circumferentia circuli in $2n$ æquales partes Ap, pB, Bq, qD, Dr, &c. & ſit P punctum in radio CA, ſi opus ſit producto, ſitum; erit $Pp \times Pq \times Pr \times Ps \times$ &c. $= a^n + 1$, & $PA \times PB \times PD \times PE \times$ &c. $= 1 - a^n$ vel $a^n - 1$.

Hic forſan haud indignum eſt obſervatu, quod chorda arcûs $l \times 360^\circ + m$, ubi m ſit affirmativa quantitas & minor quam 360° & l vel ſit par numerus vel $= o$, erit affirmativa; ſin vero l ſit impar numerus, tum prædicta chorda erit negativa quantitas: e contra ſit m negativa quantitas & minor quam 360°, tum chorda arcûs $l \times 360 - m$ erit affirmativa, ſi l ſit impar; ſin l ſit par numerus vel $= o$, tum erit negativa quantitas: etiamque in omnibus prioribus ſubſtitutionibus ſemper animadvertendum eſt; cum ſinus, coſinus, ſecans, tangens, ceteræque quantitates evadant negativæ vel affirmativæ; tum ex uno caſu generaliter invento reliqui facile inveſtigari poſſint.

THEOR. XXVII.

Fig. π. 1. Sit $ABCD$ trapezium circa conicam ſectionem deſcriptum, cujus duo latera AB & CD ſint inter ſe parallela, & cujus puncta contactuum cum conicâ ſectione ſint reſpective $\alpha, \beta, \gamma, \delta$: tum erunt $\alpha A : \gamma C :: B\alpha : \gamma D :: AB : CD$.

2. Iiſdem poſitis; & literis c, d & t reſpective pro lineis CD, AB & ſemidiametro conicæ ſectionis lineis AB & CD parallelæ, ſcriptis;

erit

erit $A\alpha = \frac{d}{2} + \sqrt{\frac{d^2 c - 4t^2 d}{4c}}$, $B\alpha = \frac{d}{2} - \sqrt{\frac{d^2 c - 4t^2 d}{4c}}$, $C\gamma = \frac{c}{d} \times A\alpha = \frac{c}{2} + \sqrt{\frac{c^2 d - 4t^2 c}{4d}}$, $\gamma D = \frac{c}{d} \times B\alpha = \frac{c}{2} - \sqrt{\frac{c^2 d - 4t^2 c}{4d}}$.

Fig. π.3. Rectæ αa, ab & βb tangant ſectionem in punctis α, δ & β; producantur lineæ αa & βb, uſque donec ſeſe interſecant in puncto B; tum erit contentum $\alpha a \times \delta b \times \beta B = a\delta \times \beta b \times \alpha B$: vel magis generaliter: (fig. ϱ.) tangant *(n)* rectæ AB, BC, CD, DE, &c. conicam ſectionem in punctis A, α, β, &c. E; producantur prima (AB) & ultima linea (ED), uſque donec ſeſe interſecant in puncto P; tum erit contentum $AB \times \alpha C \times \beta D \times$ &c. $\times EP = AP \times \alpha B \times \beta C \times$ &c. $\times ED$.

Fig. σ. Sit polygonum $ABCDE$ &c. *(n)* laterum, cujus puncta diviſionum ſint reſpective α, β, γ, δ, &c.; producantur ſingula latera, uſque donec ſeſe invicem interſecant; ſi ita dividantur hæ *(n)* lineæ in punctis diviſionum, ut *(n)* evadant diverſæ æqualitates a ſe invicem independentes duorum contentorum ejuſdem generis ac eorum in præcedente caſu vel theor. 18. traditorum; tum erunt omnia duo contenta ejuſdem generis inter ſe æqualia.

THEOR. XXVIII.

Fig. υ. 1. Sit V vertex conicæ parabolæ; ducantur lineæ VP ejus axis, $P\pi\varrho\sigma$ perpendicularis ad axem VP; $a\pi$, $bb\varrho$ & $c\sigma$ axi VP parallelæ; ak & $Vrst$ perpendiculari $P\pi\varrho\sigma$ parallela, & ac; deſignentur lineæ $P\pi$, $\pi\varrho$ & $\varrho\sigma$ per literas a, b & c; parabolæ latus rectum per literam p: tum per bene notam parabolæ proprietatem erunt $\frac{a^2}{p} = ar$, $\frac{\overline{a+b}^2}{p} = bs$, & $\frac{\overline{a+b+c}^2}{p} = ct$; ex ſimilibus triangulis cak &

bam

ham erit $ak\ (b+c) : kc = ct - ar = \frac{\overline{a+b+c}^2 - a^2}{p} :: am\ (o)$ $: mb = \frac{b+c+2a}{p} \times b$; ſed $bm + ar - bs = \frac{b+c+2a}{p} \times b +$ $\frac{a^2}{p} - \frac{\overline{a+b}^2}{p} = \frac{b \times c}{p} = bb$; & exinde area trianguli $abc = \frac{hb \times \pi\sigma}{2}$ $= \frac{\pi\varrho \times \varrho\sigma \times \pi\sigma}{2p}$.

Cor. 1. Hinc erit area trianguli abc in datâ parabolâ inſcripti in ratione contenti $\pi\varrho \times \varrho\sigma \times \pi\sigma$.

Cor. 2. Sit $\pi\varrho = \varrho\sigma = \frac{\pi\sigma}{2}$; tum cognitum eſt aream inter lineam ac & arcum abc contentam $= \frac{4}{3} \times$ aream trianguli abc; ergo erit area trianguli abc ad aream prædictam inter arcum abc & rectam ac contentam $:: \pi\varrho \times \varrho\sigma \times \pi\sigma : \frac{4}{3} \times \frac{\pi\sigma}{2} \times \frac{\pi\sigma}{2} \times \pi\sigma = \frac{1}{3} \times \pi\sigma^3$.

2. Fig. *u*. In datâ parabolâ conici generis $abcde$ &c. inſcribatur n laterum polygonum $abcde$ &c. a; ducantur lineæ $a\alpha$, $b\beta$, $c\gamma$, $d\delta$, &c. parabolæ axi reſpective parallelæ; & linea $\alpha\beta\gamma\delta$ &c. ad axem prædictum perpendicularis; tum erit area polygoni inſcripti in eâdem ratione, quam habet aggregatum $\alpha\beta \times \beta\gamma \times \alpha\gamma + \alpha\gamma \times \gamma\delta \times \alpha\delta + \alpha\delta \times \delta\varepsilon \times \alpha\varepsilon + \alpha\varepsilon \times \varepsilon\zeta \times \alpha\zeta +$ &c.

Ductis enim lineis ac, ad, ae, af, &c.; erunt per præcedentem caſum areæ triangulorum abc, acd, ade, aef, &c. contentis reſpectivis $\alpha\beta \times \beta\gamma \times \alpha\gamma$, $\alpha\gamma \times \gamma\delta \times \alpha\delta$, $\alpha\delta \times \delta\varepsilon \times \alpha\varepsilon$, $\alpha\varepsilon \times \varepsilon\zeta \times \alpha\zeta$, &c. proportionales; & conſequenter area polygoni $abcdef$ &c. a, i. e. aggregatum ſingulorum triangulorum $abc + acd + ade + aef +$ &c. erit in eâdem ratione, quam habet aggregatum $\alpha\beta \times \beta\gamma \times \alpha\gamma + \alpha\gamma \times \gamma\delta \times \alpha\delta + \alpha\delta \times \delta\varepsilon \times \alpha\varepsilon + \alpha\varepsilon \times \varepsilon\zeta \times \alpha\zeta +$ &c. ſingulorum contentorum prædictorum.

Cor.

Cor. 1. Hinc area polygoni inſcripti $abcdef$ &c. a erit ad aream inter arcum parabolæ $abcdef$ &c. & rectas lineas ab, bc, cd, de, ef, &c. contentam :: $\alpha\beta \times \beta\gamma \times \alpha\gamma + \alpha\gamma \times \gamma\delta \times \alpha\delta + \alpha\delta \times \delta\varepsilon \times \alpha\varepsilon + \alpha\varepsilon \times \varepsilon\zeta \times \alpha\zeta +$ &c. : $\frac{\alpha\beta^3 + \beta\gamma^3 + \gamma\delta^3 + \delta\varepsilon^3 + \varepsilon\zeta^3}{3}$.

Cor. 2. Sint $\alpha\beta = \beta\gamma = \gamma\delta = \delta\varepsilon = \varepsilon\zeta =$ &c. & numerus diviſionum $\alpha\beta$, $\beta\gamma$, $\gamma\delta$, $\delta\varepsilon$, $\varepsilon\zeta$, &c. ſit (n); tum erit area polygoni inſcripti $abcdef$ &c. a, ad aream inter curvam $abcde$ &c. f & rectam af contentam :: $n^2 - 1 : n^2$.

Si vero mutetur angulus, quem habet linea $\alpha\beta\gamma\delta$ &c. ad axem; eædem inter ſe manebunt rationes linearum $\alpha\beta$, $\beta\gamma$, $\gamma\delta$, $\delta\varepsilon$, &c.

Cor. Hinc conſtat theorema arithmeticum; ſint $a, b, c, d, e,$ &c. h, k, l, m, n quæcunque quantitates; tum erit

$$a \times b \times \overline{a+b} + \overline{a+b} \times c \times \overline{a+b+c} + \overline{a+b+c} \times d \times \overline{a+b+c+d} + \overline{a+b+c+d} \times e \times \overline{a+b+c+d+e} + \&c. \ldots + \overline{a+b+c+d+\&c.+h} \times k \times \overline{a+b+c+d+\&c.+h+k} + \overline{a+b+c+d+\&c.+h+k} \times l \times \overline{a+b+c+d+\&c.+h+k+l} + \overline{a+b+c+d+\&c.+h+k+l} \times m \times \overline{a+b+c+d+\&c.+h+k+l+m} + \overline{a+b+c+d+\&c.+h+k+l+m} \times n \times \overline{a+b+c+d+\&c.+h+k+l+m+n} = n \times m \times \overline{n+m} + \overline{n+m} \times l \times \overline{n+m+l} + \overline{n+m+l} \times k \times \overline{n+m+l+k} + \overline{n+m+l+k} \times h \times \overline{n+m+l+k+h+\&c.} \ldots + \overline{n+m+l+k+\&c.+e} \times d \times \overline{n+m+l+k+\&c.+e+d} + \overline{n+m+l+k+\&c.+e+d} \times c \times \overline{n+m+l+k+\&c.+e+d+c} + \overline{n+m+l+k+\&c.+e+d+c} \times b \times \overline{n+m+l+k+\&c.+e+d+c+b} + \overline{n+m+l+k+\&c.+e+d+c+b} \times a \times \overline{n+m+l+k+\&c.+e+d+c+b+a}.$$

Hoc ſequitur e ſcribendo literas $a, b, c, d,$ &c. pro lineis $\alpha\beta$, $\beta\gamma$, $\gamma\delta$, $\delta\varepsilon$, &c. in præcedentibus theorematibus.

Fig. v. 3. Sit æquatio relationem exprimens inter abſciſſam Vr & ejus correſpondentem ordinatam $ra = y = px^n$; & conſequenter $ar =$ $p \times$

$p \times Vr^n$; $bs = p \times Vs^n$; $tc = p \times Vt^n$; &c. pro lineis $rs = \pi\varrho$, $st = \varrho\sigma$ & $Vr = P\pi$ ſcribantur reſpective a, b & x; erit area trianguli abc in datâ curva inſcripti in eâdem ratione, quam habet contentum

$$\overline{a+b} \times ab \times n.\frac{n-1}{2}x^{n-2} + n.\frac{n-1}{2}.\frac{n-2}{3}x^{n-3} \times \overline{2a+b} + n.\frac{n-1}{2}.\frac{n-2}{3}.\frac{n-3}{4}x^{n-4} \times \overline{3a^2+3ab+b^2} + n.\frac{n-1}{2}.\frac{n-2}{3}.\frac{n-3}{4}.\frac{n-4}{5} \times x^{n-5} \times \overline{4a^3+6a^2b+4ab^2+b^3} + \&c.$$

Ex hinc erui poſſit expreſſio pro detegendâ areâ polygoni in datâ curvâ inſcripti: datâ generali expreſſione pro quolibet triangulo in datâ curvâ inſcripto, ex additione conſtat expreſſio pro areâ polygoni in datâ curvâ inſcripti; polygonum enim in triangula facile dividi poteſt.

Principiis in præcedente theoremate arithmetico traditis, ad hunc caſum applicatis; exinde deduci poſſint plura conſimilia theoremata arithmetica.

THEOR. XXIX.

Fig. *M.* Sit hyperbola MAn, cujus aſymptoti ſint aC & Cb, axis vero AM, & ejus vertices M & A, & linea mn ordinata ad axem, & ſit v quodlibet datum punctum in ordinatâ mn contentum: a dato puncto (v) ducantur (ſi modo poſſibile ſit) quatuor lineæ perpendiculares in oppoſitas hyperbolas in punctis e, f, g, h: ex his punctis ducantur lineæ $e\alpha$, $f\beta$, $g\gamma$, $h\delta$ ordinatæ ad axem. Ducantur etiam a dato puncto (v) duo perpendicula (vH & vI) ad aſymptotos: e punctis H & I ducantur lineæ Hr & Is perpendiculares ad axem datæ hyperbolæ: erit ſumma vel differentia linearum ($e\alpha$, $f\beta$, $g\gamma$, $h\delta$), prout ex eâdem vel oppoſitis axis partibus incidant, viz. $e\alpha \pm f\beta \pm g\gamma \pm hd = \pm 2 \times \overline{Hr - Is}$: & eodem modo ſumma vel differentia linearum

linearum ($A\alpha$, $A\beta$, $A\gamma$, $A\delta$,) prout ex eâdem vel diverſis partibus verticis (A) incidant, viz. $A\alpha \mp A\beta \mp A\gamma \mp A\delta = 2\overline{Ar \pm As}$.

Idem affirmari poſſit mutatis mutandis de lineis ſimiliter deductis e lineis ſecantibus curvam & ejus aſymptotos in quolibet dato angulo.

Cor. 1. Quantitas $A\alpha \pm A\beta \pm A\gamma \pm A\delta$ ſemper eadem erit, cum punctum v ponitur in eâdem ordinatâ mn.

Cor. 2. Quantitas $e\alpha \pm f\beta \pm g\gamma \pm h\delta$ ſemper eadem manet, cum punctum v ponitur in lineâ vx hyperbolæ axi parallelâ.

Cor. 3. Contentum ſub quatuor ordinatis $e\alpha \times f\beta \times g\gamma \times h\delta$ idem erit, cum punctum v ad lineam vx axi parallelam referatur: & contentum $A\alpha \times A\beta \times A\gamma \times A\delta$ ſemper idem erit, cum punctum v in eâdem axis ordinatâ ſit poſitum.

Hæc corollaria etiam affirmari poſſint de quibuſlibet quatuor perpendiculis e dato puncto in ellipſin incidentibus.

Hoc theorema ſequitur e proprietatibus algebraicarum curvarum in theor. xii. traditis.

SCHOLIUM.

Proprietates conicarum ſectionum prius traditæ ex his duobus principiis a me primum excogitatis potiſſimum conſequuntur: ſcilicet 1mo quod quantitates, quæ ad ſingulum curvæ punctum recipiant maximum vel minimum, perpetuo evadunt inter ſe æquales. e. g. fig. 64. angulus (ACB) in curvæ ſegmento (ACB) erit maximus; cum angulus BCF, quem linea CF tangens curvam ad punctum C facit cum lineâ BC, æqualis ſit angulo BAC; & conſequenter incrementum anguli ACB nihilo ſit æquale: ſi vero incrementum hujus anguli in dato ſegmento contenti perpetuo nihilo ſit æquale, i. e. angulus prædictus perpetuo ſit maximus, vel $\angle BCF = \angle BAC$; tum angulus in dato ſegmento contentus ſemper idem erit; hoc valet in circulo:

fig.

fig. 62. eodem modo, ſumma linearum *(AF* & *AH)* a duobus punctis (*F* & *H*) ad curvam ductarum erit maxima, cum anguli *FAQ* & *HAR*, quos utrinque cum tangente faciunt lineæ *AF* & *AH*, ſint inter ſe æquales: lineæ a duobus focis *(F* & *H)* ad quodlibet punctum *A* ellipſeos ductæ, æqualem angulum *(FAQ* & *HAR)* utrinque cum tangente faciunt; ergo ſemper ſumma $FA + HA$ erit data quantitas. Et vice versâ in omni caſu; ſi quantitates, e. g. lineæ, utcunque mutentur earum inclinationes, &c. ſemper ſint datæ quantitates; tum hæ lineæ, &c. gaudent proprietatibus, quæ ad earum maximum vel minimum pertinent, e. g. ſumma linearum a duobus focis ad punctum ellipſeos ductarum eſt data quantitas; cum ſumma linearum ita ductarum ſit maxima quantitas, tum æquales angulos utrinque cum tangente curvæ faciunt prædictæ lineæ; ergo æquales angulos in ellipſi cum tangente faciunt lineæ a duobus ejus focis ductæ.

2do. Sint duæ æquationes (n & m dimenſionum) duas incognitas quantitates x & y habentes; reducantur hæ duæ æquationes in unam, ita ut exterminetur altera incognita quantitas (x): ſi æquatio reſultans aſcendat ad nm dimenſiones; ſumma ejus radicum pendet e terminis, in quibus inveniuntur n & $n - 1$ dimenſiones in unâ æquatione, m & $m - 1$ dimenſiones in alterâ; & ſit deinceps; & exinde, ſi modo ſint duæ aliæ æquationes n & m dimenſionum, quæ involvant quantitates z & v; & ſint termini in his reſpectivis æquationibus, in quibus inveniuntur dimenſiones n & $n - 1$, m & $m - 1$ reſpective, iidem ac in duabus prædictis æquationibus; tum ſumma omnium valorum quantitatum x & y eadem erit ac ſumma omnium valorum quantitatum z & v.

PROPRIETATES.

THEOR. IX. 2. Pag. 45.

Fig. II. Sit æquatio algebraica n dimenſionum relationem inter abſciſſam (AP, x) & ejus correſpondentes ordinatas (PM, y) deſignans; & in eâ contineantur ſolummodo termini Px^{n-p}, Qx^{n-q}, Rx^{n-r}, &c. dimenſionum $n-p$, $n-q$, $n-r$, &c. gyretur recta AM circa punctum A, ſecans curvam in (n) punctis M, M', M'', &c. ad puncta M, M', M'', &c. agantur ordinatæ parallelæ PM, $P'M'$, $P''M''$, &c. tum invariabilis erit omnis functio e literis A, B, C, D, &c. conflata (ubi literæ A, B, C, D, &c. reſpective denotant aggregata e ſingulis p, q, r, &c. abſciſſis AP, AP', AP'', &c. in ſeſe ductis) quæ ſit fractio, cujus in ſingulis terminis & numeratoris & denominatoris eædem dimenſiones quantitatum $(A, B, C, D$, &c.) conjunctim habentur; ſi modo dimenſiones contenti $A^\alpha \times B^\beta \times C^\gamma \times D^\delta \times$ &c. per ſummam $\alpha + \beta + \gamma + \delta +$ &c. denotentur.

Scribatur enim in datâ æquatione pro y ejus valor σx, ubi $\sigma = \frac{\text{ſin.} \angle PAM}{\text{ſin.} \angle PMA}$; reſultat æquatio, cujus radices ſint AP, AP', AP'', &c. deinde transformetur fractio prædicta in alteram invariabiles quantitates P, Q, R, &c. ſolummodo involventem; hæc fractio erit invariabilis, utcunque varietur litera σ.

Fig. P. Ex. Sit curva conica ſectio, cujus abſciſſa (AP) incipiat a quolibet puncto A in diametro (AP) poſito; gyretur recta AM circa punctum A, ſecans conicam ſectionem in duobus punctis M & M'; agantur ordinatæ MP & $M'P'$; tum erit $\frac{AP \times AP'}{\pm AP \pm AP'}$ invariabilis quantitas, utcunque mutetur angulus MAP.

1. 2. In æquatione algebraicâ n dimenſionum prædictâ contineantur ſolummodo termini Py^{n-p}, Qy^{n-q}, Ry^{n-r}, &c. dimenſionum $n-p$, $n-q$, $n-r$, &c. reſpective; tum invariabilis erit omnis functio e

literis

literis *A*, *B*, *C*, *D*, &c. conflata (ubi literæ *A*, *B*, *C*, *D*, &c. reſpective denotant aggregata e ſingulis *p*, *q*, *r*, &c. ordinatis in ſeſe ductis) quæ ſit fractio ejuſdem generis ac ea in præcedente caſu tradita.

1. 3. In prædictâ algebraicâ *n* dimenſionum æquatione contineantur ſolummodo termini $P \times \overline{x^2+y^2}^{\frac{n-p}{2}}$, $Q \times \overline{x^2+y^2}^{\frac{n-q}{2}}$, $R \times \overline{x^2+y^2}^{\frac{n-r}{2}}$, &c. dimenſionum $n-p$, $n-q$, $n-r$, &c. tum invariabilis erit omnis functio præcedentis generis e literis *A*, *B*, *C*, *D*, &c. conflata, ubi literæ *A*, *B*, *C*, *D*, &c. reſpective denotant aggregata e ſingulis *p*, *q*, *r*, &c. lineis *AM*, *AM'*, *AM''*, &c. in ſeſe ductis, & $\frac{n-p}{2}$, $\frac{n-q}{2}$, $\frac{n-r}{2}$, &c. integri ſint numeri; & ſic deinceps.

2. Sit y^n terminus æquationis prædictæ, in quo maximæ (*n*) inveniantur dimenſiones, etiamque ſint Py^{n-p}, Qy^{n-q}, Ry^{n-r}, &c. termini, in quibus ſolummodo contineantur dimenſiones $n-p$, $n-q$, $n-r$, &c. reſpective; tum omnis data functio aggregatorum e ſingulis *p*, *q*, *r*, &c. ordinatis *PM*, *PM'*, *PM''*, &c. in ſeſe ductis, i. e. e literis *P*, *Q*, *R*, &c. erit invariabilis quantitas, utcunque varietur angulus *PAM*. Hoc facile conſtabit ſcribendo pro *x* in datâ æquatione πy, ubi $\pi = \frac{\text{ſin.} \angle PMA}{\text{ſin.} \angle PAM}$. Fig. ϱ. 1. Sit curva parabola conici generis, cujus abſciſſa ſit diameter, & ab ejus puncto *A* incipiat; per punctum *A* agatur linea *MAM'* ſecans parabolam in punctis *M* & *M'*; ad puncta *M* & *M'* agantur ordinatæ *MP* & *M'P'*; tum erit rectangulum $MP \times M'P' = AR^2$ quadrato diametri ordinatæ per punctum *A* tranſeuntis. 2. Sit *AR* linea tangens parabolam in puncto *R*, ad punctum *R* ducatur *Rr* ordinata diametri *AP*; agatur linea *AM* ſecans parabolam in punctis *M* & *M'*, etiamque *PM* & *P'M'* ordinatæ diametri *AP*; tum erit $PM \times P'M' = Rr^2$.

2. 2. Si pro y^n ſcribatur x^n & pro Py^{n-p}, Qy^{n-q}, Ry^{n-r}, &c. in prædictâ algebraicâ æquatione ſcribantur Px^{n-p}, Qx^{n-q}, Rx^{n-r}, &c. tum omnis functio aggregatorum e ſingulis *p*, *q*, *r*, &c. abſciſſis *AP*, *A'P'*, *A''P''*,

$A''P''$, &c. in ſeſe ductis in curvâ reſultante, i. e. e literis P, Q, R, &c. erit invariabilis, utcunque mutetur angulus PAM.

2. 3. Sit $\overline{x^2+y^2}^{\frac{n}{2}}$ terminus prædictæ algebraicæ æquationis, in quo maximæ contineantur dimenſiones; & $P\overline{y^2+x^2}^{\frac{n-p}{2}}$, $Q\times\overline{y^2+x^2}^{\frac{n-q}{2}}$, $R\times\overline{y^2+x^2}^{\frac{n-r}{2}}$, &c. termini, in quibus ſolummodo contineantur $n-p$, $n-q$, $n-r$, &c. dimenſiones; tum omnis functio aggregatorum e ſingulis p, p, r, &c. lineis AM, AM', AM'', &c. in ſeſe ductis, erit eadem quantitas, utcunque varietur angulus PAM. Hoc facile deduci poteſt ſcribendo z, ϱz & τz; pro $\overline{x^2+y^2}^{\frac{1}{2}}$, x & y in datâ algebraicâ æquatione, ubi $\varrho=\frac{\text{ſin.}\angle PMA}{\text{ſin.}\angle APM}$, & $\tau=\frac{\text{ſin.}\angle PAM}{\text{ſin.}\angle APM}$.

Fig. 70. Pag. 56. lin. 24.

2. Iiſdem lineis ductis; ſit $y^m=ax^n+bx^{n-1}+cx^{n-2}+$ &c. æquatio relationem inter abſciſſam $A\pi$ & ejus correſpondentem ordinatam $\pi\mu$ deſignans; tum erit correſpondens earum ſubtangens $\frac{y\dot{x}}{\dot{y}}=m\times\frac{ax^n+bx^{n-1}+cx^{n-2}+\&c.}{nax^{n-1}+\overline{n-1}\,bx^{n-2}+\&c.}$; at $ax^n+bx^{n-1}+cx^{n-2}+$ &c. $=a\times\pi a\times\pi b\times\pi c\times\pi d\times$ &c.; & $nax^{n-1}+\overline{n-1}\,bx^{n-2}+$ &c. $=n\times a\times\pi P\times\pi Q\times\pi R\times$ &c. unde $\pi P\times\pi Q\times\pi R\times$ &c. : $\pi a\times\pi b\times\pi c\times\pi d\times$ &c. :: $\frac{m}{n}$: ſubtangentem.

Correſpondens earum ſubnormalis $\frac{y\dot{y}}{\dot{x}}=\frac{1}{m}\times\overline{ax^n+bx^{n-1}+cx^{n-2}+\&c.}^{\frac{2-m}{m}}\times\overline{nax^{n-1}+\overline{n-1}\,bx^{n-2}+\overline{n-2}\,cx^{n-3}+\&c.}$ unde $(\pi a\times\pi b\times\pi c\times\pi d\times\&c.)^{\frac{m-2}{m}}$: $\pi P\times\pi Q\times\pi R\times$ &c. :: $\frac{n}{m}\times a\,\frac{2}{m}$: ſubnorm.

Et ſic facile deduci poſſint quam plurimæ conſimiles propoſitiones.

THEOR. XXX.

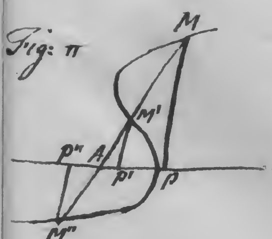
Fig. Ω. Sint tres rectæ abd, bce & acf; aſſumatur $al = \frac{ab(b) \times bg(z) + cb(d) \times \text{coſ.} \angle cba(c) \times z - d \times ba(a)}{z - 2d}$, ubi cb eſt perpendiculum a puncto c in rectam ab demiſſum; agatur linea ag; tum diameter trium rectarum, cujus ordinata ſit ag, per punctum l tranſit.

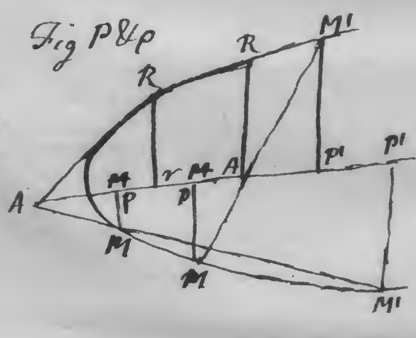
Demittatur enim gm perpendicularis ad bm, & agatur recta ck parallela ordinatæ ga; aſſumantur $ap = \frac{ag}{3}$ & $kn = \frac{2}{3} \times ck$: recta per puncta p & n tranſeuns, erit diameter ad ordinatas parallelas ga & ck: patet $ga^2 = gm^2 + am^2 = z^2 - 2caz + a^2 = B^2$; unde ob ſimilia triangula agb & kcb erit $z : B :: d : ck = \frac{d \times B}{z}$; etiamque ob ſimilia triangula gma & cbk, $B : ma = mb - ab = cz - a :: ck = \frac{d \times B}{z} : bk = \frac{d}{z} \times \overline{cz - a}$; ultimo, ſi modo agatur linea qn parallela lineæ ab, ob ſimilia triangula pnq & pla erit $pq = \frac{1}{3} ga - \frac{2}{3} ck = \frac{z - 2d}{3z} B : pa = \frac{B}{3} :: qn = ab + bk = b + \frac{d}{z} \times \overline{cz - a}$ $\frac{bz + cdz - da}{z} : al = \frac{b \times z + d \times c \times z - d \times a}{z - 2d}$. Q. E. D.

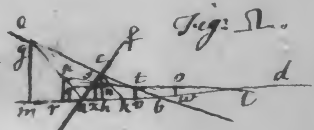
Cor. Hinc erui poteſt æquatio relationem exprimens inter abſciſſam (x) & ejus correſpondentes ordinatas (y) curvæ, cujus puncta conſtituunt locum centrorum prædictarum trium rectarum.

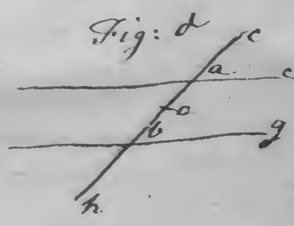
Secet recta pnl duas rectas af & be in punctis s & t; demittantur lineæ [illegible], sx, tv & ow perpendiculares ad rectam ab; ubi o ſit centrum trium rectarum, cujus locus requiritur: ob quatuor ſimilia triangula

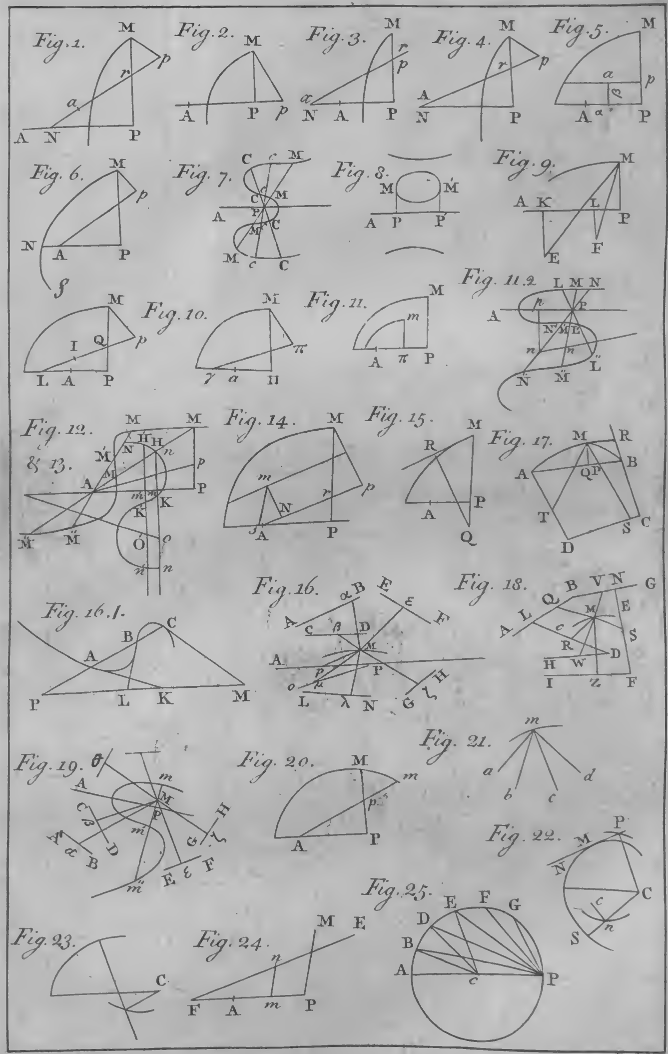
triangula lpr, lsx, ltv & low erunt $al+\frac{1}{3}ma\left(\frac{cz-a}{3}\right):pr=\frac{gm}{3}=\frac{\sqrt{1-c^2}\times z}{3}::lx=al-ax:sx=\frac{\text{ſin.}\angle cab}{\text{coſ.}\angle cab}\times ax=\pi\times ax$; & $al+\frac{1}{3}ma:pr::lv=al-ab+bv:tv=\frac{\sqrt{1-c^2}}{c}\times bv=\varrho\times bv$; & $al+\frac{1}{3}ma:pr::lw=al-aw\,(x):ow\,(y)$; unde $\overline{al+\frac{1}{3}ma}\times y=pr\times\overline{al-x}$: at, quoniam o eſt centrum, evadet $so+to=ol$; & conſequenter $\overline{xw}+\overline{vw}=\overline{aw-ax}\,(xw)+\overline{aw-ab+bv}\,(vw)=lw=al-aw=al-x$. Scribantur pro lineis ax & bv in duabus æquationibus ($\overline{al+\frac{1}{3}ma}\times y=pr\times\overline{al-x}$ & $3x-a-ax+bv=al$) earum valores $\frac{pr\times al}{\pi\times al+\frac{1}{3}\pi\times ma+pr}$ & $\frac{pr\times\overline{al-ab}}{\varrho\times al+\frac{1}{3}\varrho\times ma-pr}$; & pro lineis pr, al & ma earum valores prius traditi; reſultant duæ æquationes tres incognitas quantitates x, y & z habentes; reducantur hæ duæ æquationes in unam, ita ut exterminetur quantitas z; exorietur æquatio relationem inter abſciſſam x & ejus correſpondentes ordinatas y deſignans. Fig. ȣ. Sint duæ rectæ af & bg inter ſe parallelæ, ſecet tertia recta hc duas parallelas in punctis a & b, biſecetur linea ab in o; tum punctum o erit centrum generale trium rectarum af, bg & hc.










Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.
Fig. 10.
Fig. 11.
Fig. 11.2
Fig. 12. & 13.
Fig. 14.
Fig. 15.
Fig. 17.
Fig. 16.
Fig. 16.1.
Fig. 18.
Fig. 19.
Fig. 20.
Fig. 21.
Fig. 22.
Fig. 25.
Fig. 23.
Fig. 24.

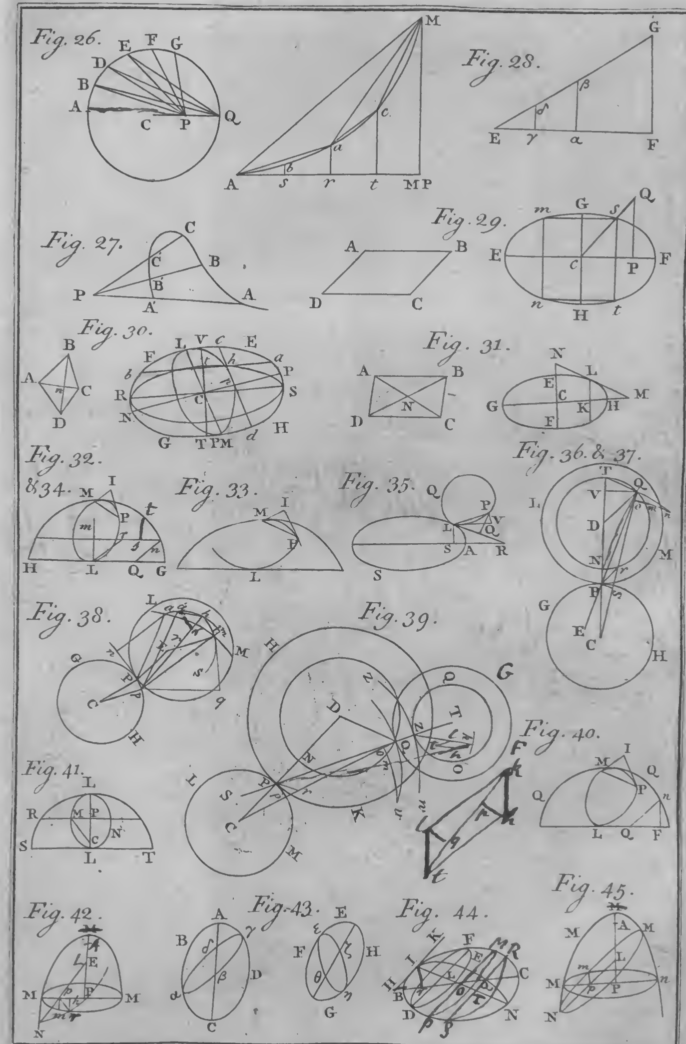

Fig. 26.
Fig. 27.
Fig. 28.
Fig. 29.
Fig. 30.
Fig. 31.
Fig. 32. & 34.
Fig. 33.
Fig. 35.
Fig. 36. & 37.
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Fig. 42.
Fig. 43.
Fig. 44.
Fig. 45.

Fig. 46.

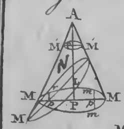


Fig. 47.

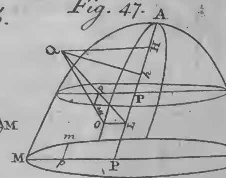


Fig. 48.

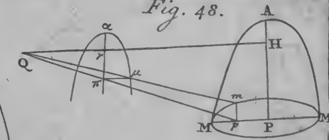


Fig. 49.

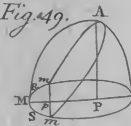


Fig. 50.

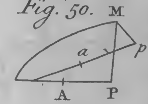


Fig. 51.

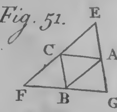


Fig. 52.

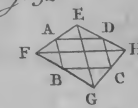


Fig. 53.

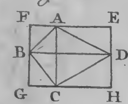


Fig. 54.

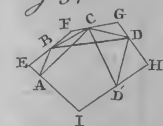


Fig. 55.

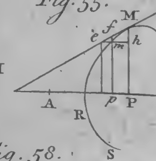


Fig. 57.

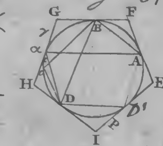


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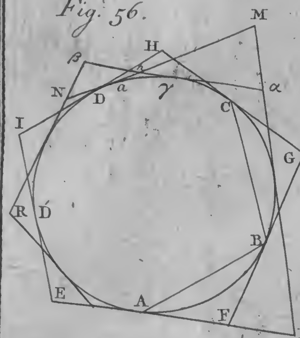


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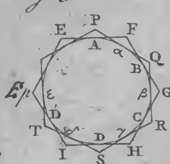


Fig. 57. 2

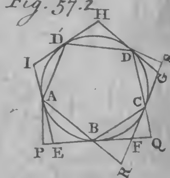


Fig. 59.

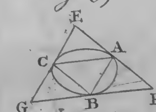


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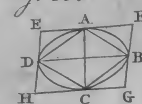


Fig. 61.



Fig. 62.

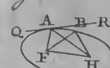


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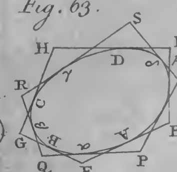
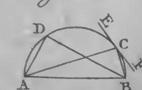


Fig. 64.



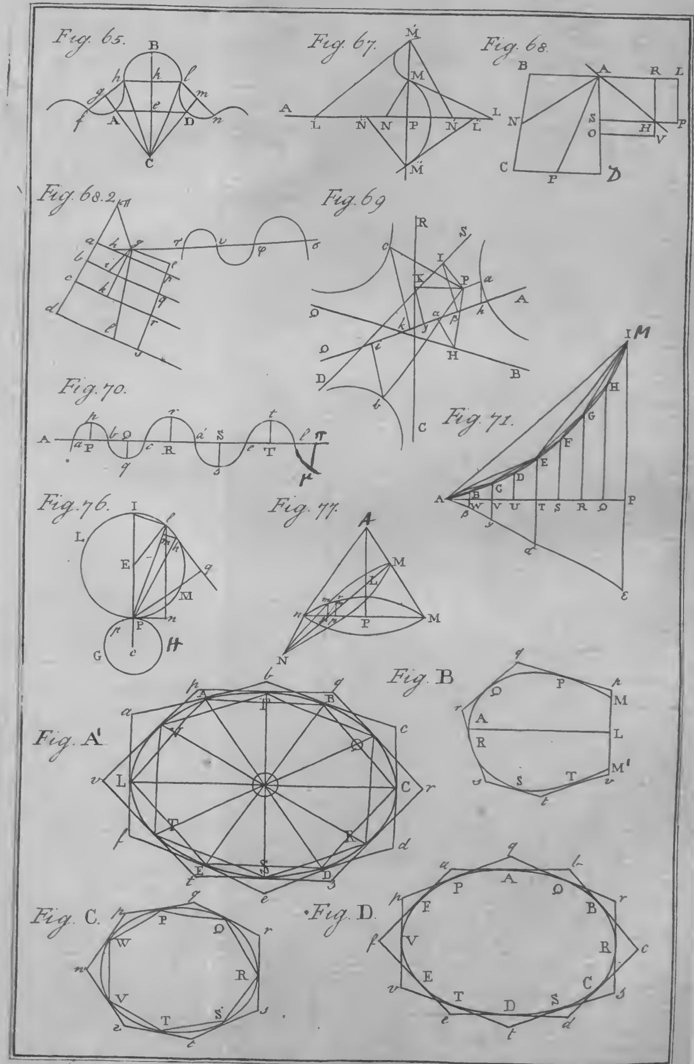


Fig. 65.
Fig. 67.
Fig. 68.
Fig. 68.2
Fig. 69
Fig. 70.
Fig. 71.
Fig. 76.
Fig. 77.
Fig. B
Fig. A'
Fig. C.
Fig. D.

Fig. A.

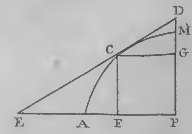


Fig. a

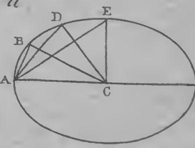


Fig. b.

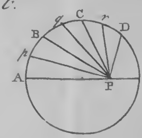


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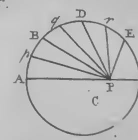


Fig. M.

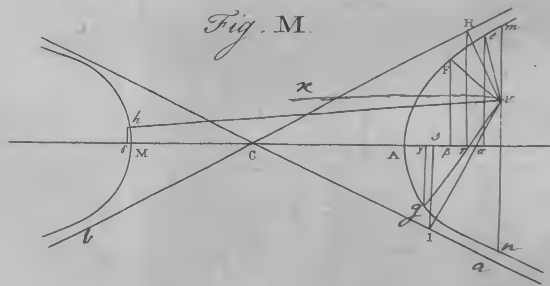


Fig. E.

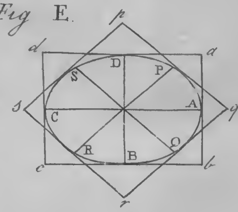


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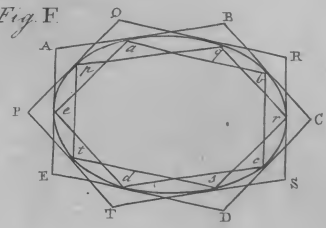


Fig. G

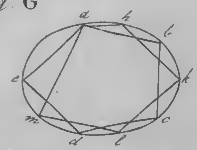


Fig. H

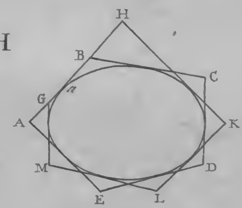


Fig. I.

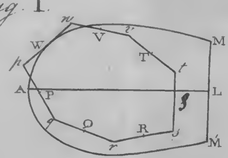
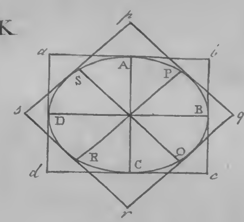


Fig. K



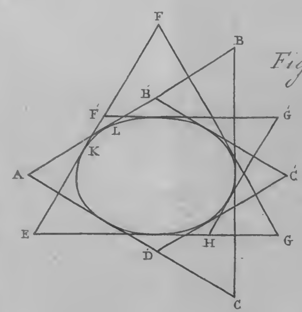
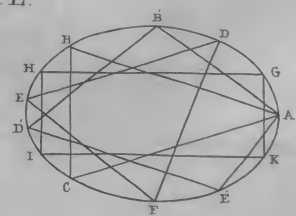


Fig. L.



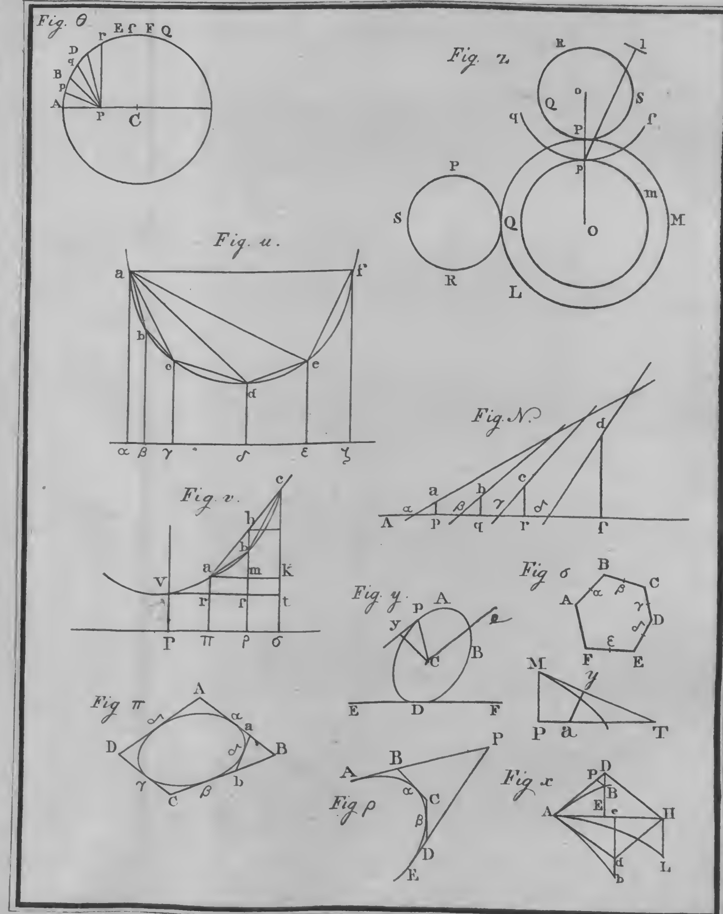


Fig. θ
Fig. z
Fig. u.
Fig. N.
Fig. v.
Fig. y.
Fig σ
Fig π
Fig ρ
Fig x